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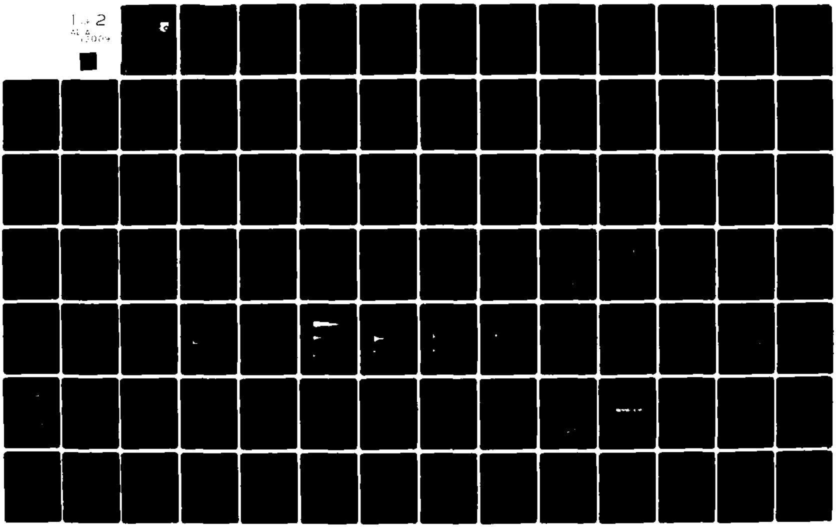

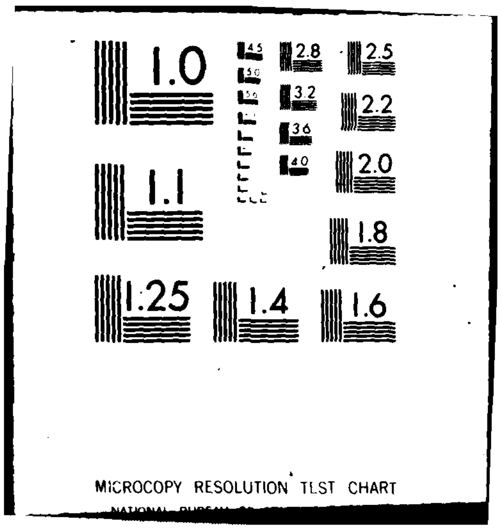
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AFWAL-TR-81-4148

INTERACTIVE GRAPHIC SIMULATION OF ROLLING ELEMENT BEARINGS
Phase I: Low Frequency Phenomenon and RAPIDREB Development

Mechanical Technology Incorporated
968 Albany-Shaker Road
Latham, New York 12110

November 1981

FINAL REPORT FOR PERIOD AUGUST 1980 - SEPTEMBER 1981

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This technical report has been reviewed and is approved for publication.

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A selective suppression of the very high frequency content of the generalized motion simulated by the original DREB computer program has led to a considerable increase in the maximum permissible time step size and hence performance simulations over relatively large time domains (several shaft revolutions) have been economically possible with the updated version RAPIDREB. Capabilities of RAPIDREB are demonstrated for both light load, low speed DMA (Despun Mechanical Assembly) ball bearings and the heavy load, high speed

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) engine type ball bearings. From the simulations obtained over a shaft revolution it is shown that the race guided cage in the DMA bearings is generally stable while the ball guided cage produces relatively noisy and to some extent unstable cage motion. The high speed engine bearing performance is simulated over more than seven shaft revolutions. The steady whirl and continued contact of the cage with the guiding race is simulated under a combined thrust and radial load applied on the bearing. Thus conditions of excessive cage wear and eventual failure are simulated.

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1.0 INTRODUCTION

With the availability of advanced computer codes, such as the Dynamics of Rolling Element Bearings (DREB) computer program [1-5], the interest in real time performance simulation of rolling bearings has been growing rather rapidly over the past few years. The most generalized dynamic model underlining DREB has been employed for the purpose of both design and performance diagnosis in advanced rolling bearing systems. However, in spite of the rapidly advancing computer technology, the computational effort required in using DREB has proven to be a practical limitation. This is related directly to the nature of the model in the sense that the maximum step size for the numerical integration of a system of differential equations of motion is determined by the highest frequency present in the system, which is often several orders of magnitude higher than the shaft rotational speed. Thus in order to simulate effects of the order of shaft speed, integration over a very large time domain is required and since the maximum step size is limited by the high frequencies, the computing costs, specially for very low speed applications, have proven to be prohibitive. The primary objective of the present investigation is to examine the various time scales in the general behavior of a ball bearing and accordingly modify DREB to suppress the very high frequencies and thereby develop a "rapid version" of DREB, which has been called RAPIDREB.

For a typical ball bearing the high frequency present in the system is defined by the ball/race Hertzian contact. If the ball is free to translate, vibratory motion corresponding to this natural frequency has been observed. Also, as will be discussed in the next section, a relatively high frequency is also associated with the kinematics of ball motion. In order to suppress these high frequencies the ball mass center may be constrained in a suitable manner. Once this is done the step size can be made relatively large and also predictor-corrector type algorithms can be employed to considerably reduce the computing costs. However, if a cage is present in the bearing, the discontinuities associated with the ball/cage and race/cage collisions can produce stability problems for the predictor-corrector schemes and therefore such algorithms can only be effectively used in the case of a cageless bearing, and considerable care will be required in using such algorithms when there are repeated ball/cage or race/cage collisions. In general it will be necessary to re-start the predictor-corrector process at the time of the start and end of such collisions if the collision time is large. For short collisions an explicit formula can be used during the entire collision. The ultimate integrating scheme may therefore be of a hybrid nature employing both explicit and implicit algorithm. Thus, the incorporation of both types of formulae has been one of the objectives of the present work.

The next section of this report reviews the various time scales associated with generalized motion of the rolling elements in a ball bearing. Section 3 contains the essential modifications to the existing program DREB in order to develop the RAPIDREB version. Some dynamic simulations for both the very low speed DMA* bearings and high speed engine bearings are then presented as typical test cases demonstrating the capabilities of RAPIDREB. Descriptions of program inputs are presented in Appendix A, and typical outputs of the program are shown in Appendices B and C.

* Despun Mechanical Assembly

2.0 GENERAL TIME SCALES

The characteristic frequencies in the general dynamic performance of ball bearings normally cover a wide spectrum. Frequencies associated with the elastic contact phenomenon are on the high end while the shaft speed is on the low end of the spectrum. Fortunately for most operating environments the high frequencies are several orders of magnitude greater than the frequencies of the order of shaft revolution speed and therefore it is relatively easy to constrain the motion to eliminate all the high frequencies in order to look at the low frequency phenomenon in depth. To understand the physical mechanism behind each frequency the basic model for each interaction and the fundamental kinematics of ball motion must be reviewed.

2.1 Ball/Race Interaction

The normal contact loads at the ball/race interaction are computed by the Hertzian elastic contact theory. For small changes in load the non-linear Hertzian load-deflection relation can be suitably linearized.
Now if the ball is free to translate arbitrarily a vibration frequency corresponding to this spring will be observed. This has been discussed in depth by Gupta et al [6]. To review the phenomenon briefly Figure 2-1 shows typical axial and radial components of ball mass center as it travels in its orbit. The high frequency component corresponds to the elastic contact phenomenon. For most ball bearings this frequency can be in the range of 1 to 50 kHz. A large bearing will normally have a lower elastic contact frequency due to heavier balls.

Figure 2-1 also shows a distinct low frequency whose axial and radial components are 180° out of phase. This frequency has been termed as a kinematic frequency [6] since it is a strong function of race curvatures and it is proportional to square root of ball/race contact load. Again the actual frequency for a given bearing depends on the geometry of bearing and the applied conditions but the general range is from about 500 Hz to 10 kHz.

It is interesting to review the load dependence of both the above frequencies. The kinematic frequency, by definition, has a square root variation as dis-

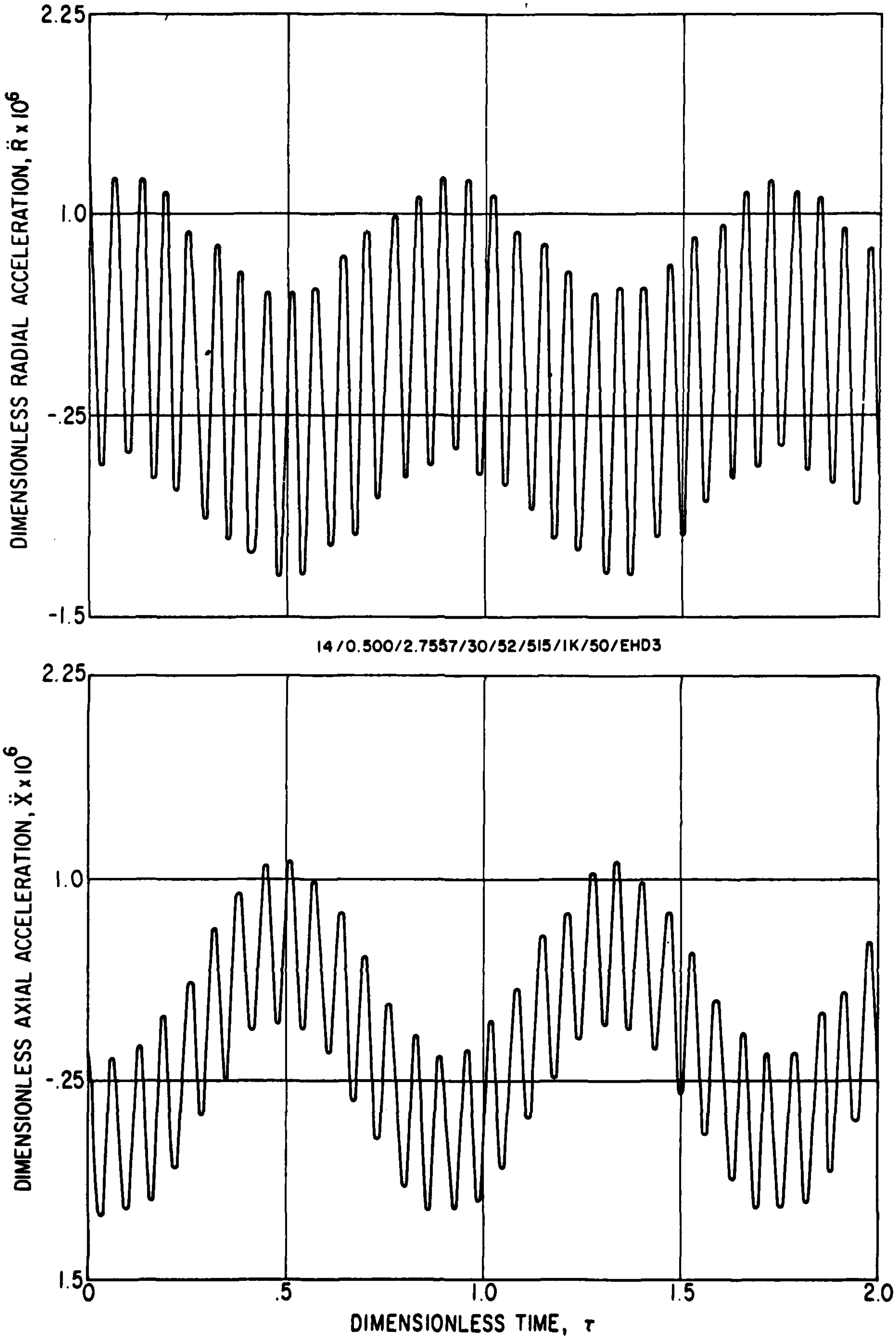

Fig. 2-1 Characteristic Ball Mass Center Vibration Pattern in an Angular Contact Ball Bearing

cussed above and the load dependence of the elastic contact frequency can be easily shown as 1/6 power os load:

Hertz contact load $Q = K\delta^{3/2}$

Stiffness $k = \frac{\partial Q}{\partial \delta} = \frac{3}{2} K \delta^{1/2} = \frac{3}{2} K \left(\frac{Q}{K}\right)^{2/6}$

Natural frequency $= \sqrt{\frac{k}{m}} \sim Q^{1/6}$

Figure 2-2 shows such a dependence for a ball bearing with 12.7 mm diameter balls and 70 mm pitch diameter. It is seen that as the contact load increases the two characteristic frequencies come close together. Thus either at high applied loads or at high speeds, the distinction between the two frequencies may not be as clearly seen in the ball acceleration plots as it is seen in Figure 2-1.

2.2 Ball/Cage and Race/Cage Interactions

The dynamics of ball/cage interaction can be quite complicated depending on the operating conditions and the ball pocket clearances. For a simple thrust loaded condition with gravity acting along the bearing axis, the cage weight will be supported uniformly by all the balls and therefore ball/cage contact will be established in each pocket. For such a condition the elastic contact spring modeling the normal contact will produce a vibratory motion if the cage is allowed to translate axially relative to the balls. Such a phenomenon is demonstrated in Figure 2-3. Again the frequencies will be generally high compared to the shaft rotational speed.

In most instances the ball/cage contacts may be of impulsive nature and the total time of contact may be very small. A typical example is presented in Figure 2-4. Such impulsive contacts will produce definite discontinuities in the motions of both the balls and the cage and considerable care will be required in integrating the equations of motion.

2.3 Low Frequency Components

This class of motion basically contains frequencies of the order of race angular velocities. The components of interest generally include ball

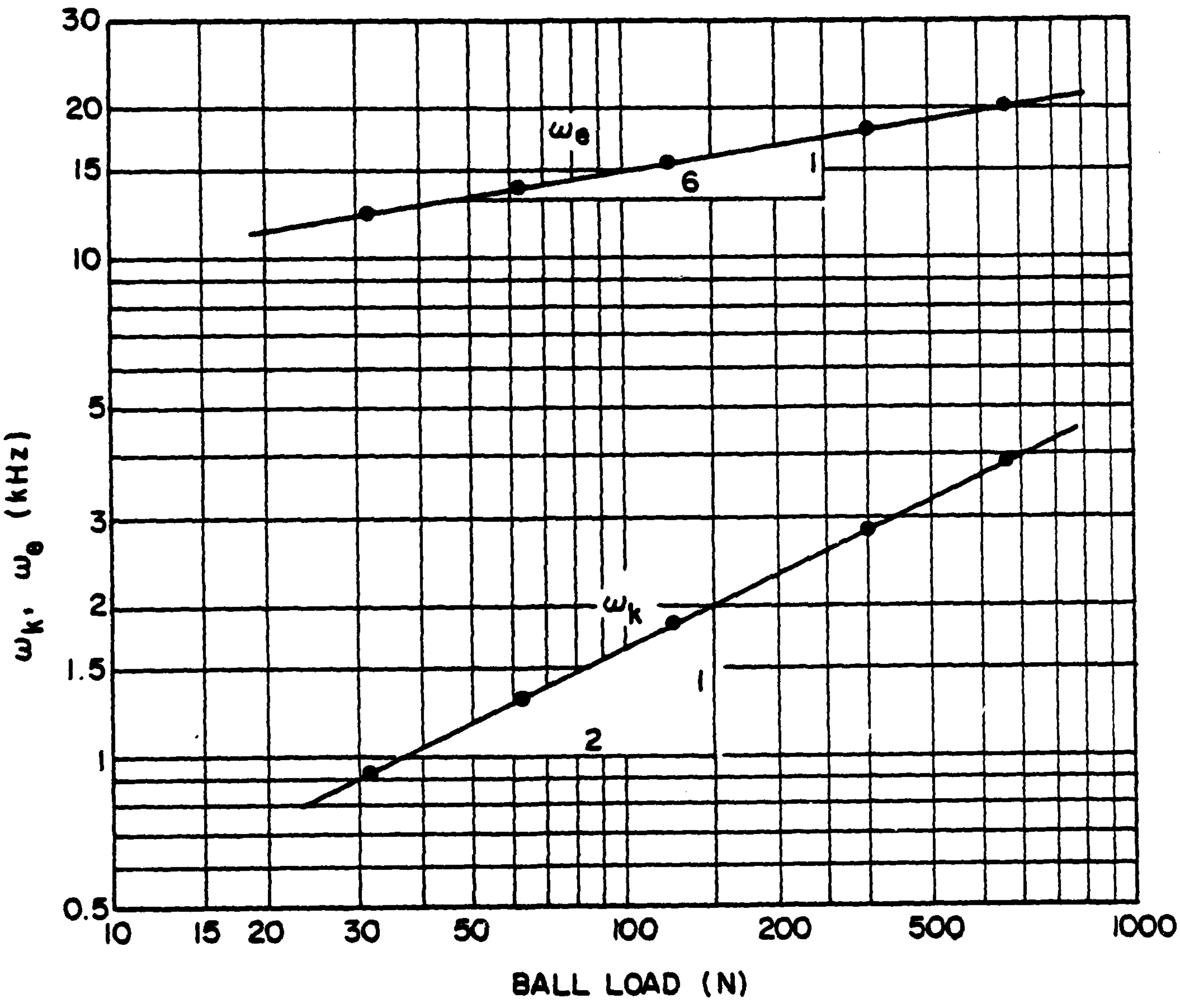

Fig. 2-2 Variations of computed elastic contact frequency, ω_e and bearing kinetic frequency, ω_k as a function of ball-race contact load

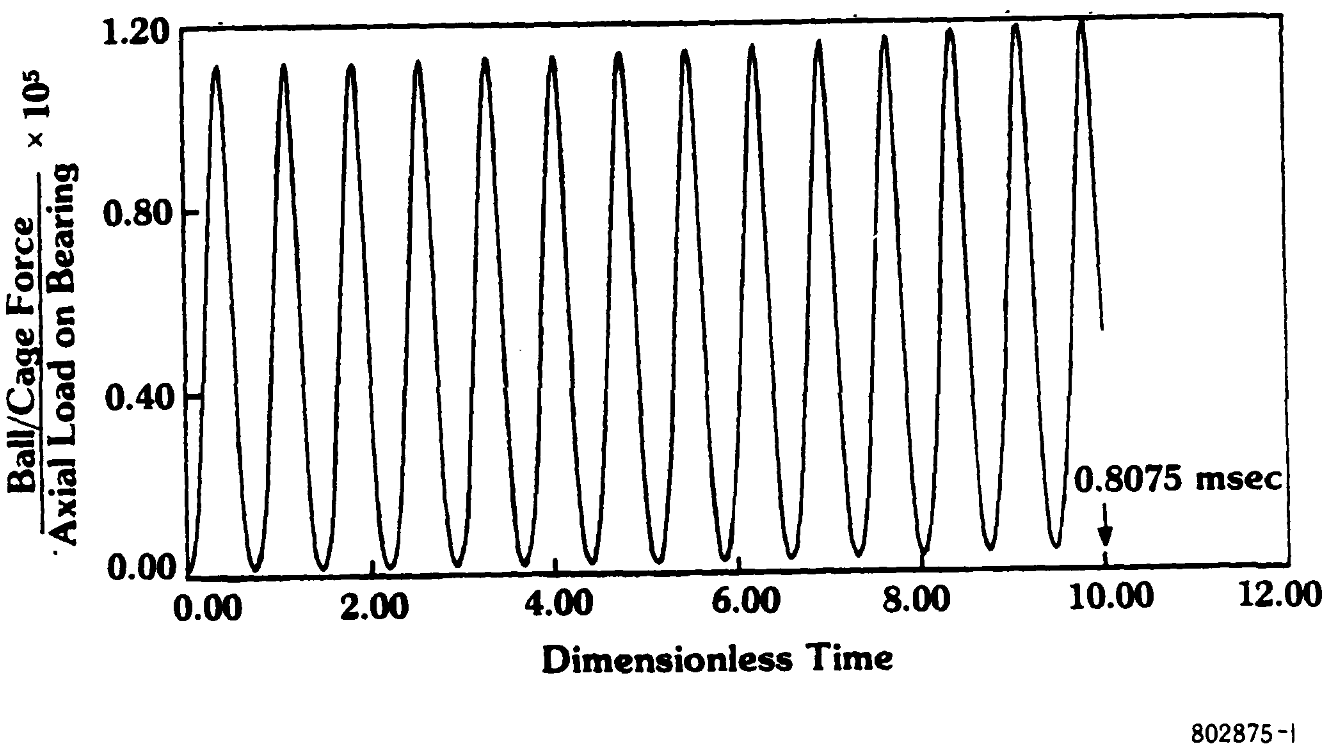

Fig. 2-3 Typical Ball/Cage Contact Vibration

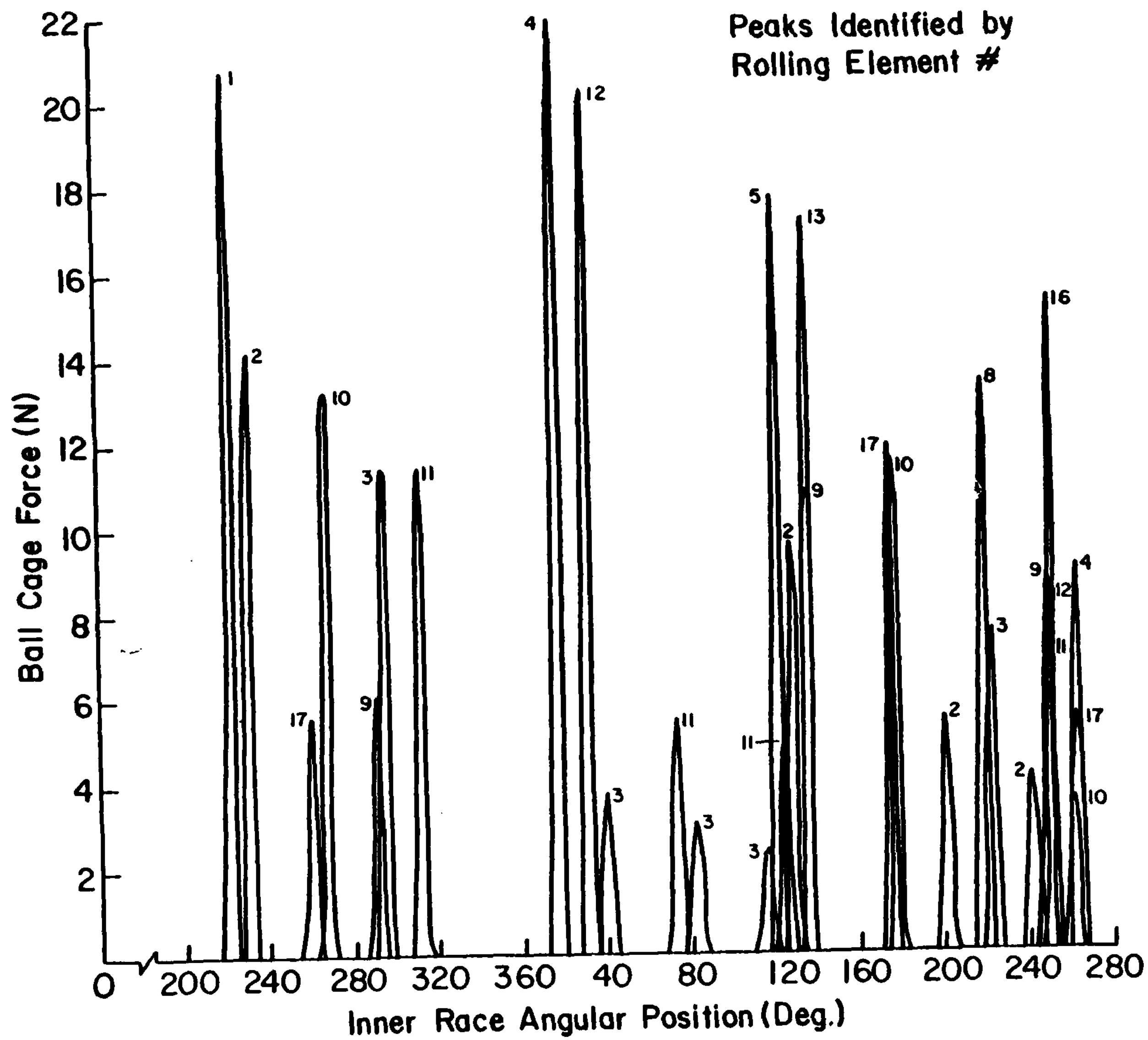

Fig. 2-4 Typical Ball/Cage Collisions in an Angular Contact Ball Bearing with Combined Axial and Radial Load

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angular velocity, ball orbital velocity, ball pass frequency (rate at which a given point on the race is passed by the balls) etc. All of these frequencies may not be constant depending on the operating conditions. In fact, the variation in ball angular velocity may sometimes be quite large. This can be easily illustrated if the equations of ball angular motion are written in a coordinate frame rotating with a velocity $\dot{\theta}$ which is also the ball orbital velocity:

$$
\begin{aligned}
I\dot{\omega}_1 &= G_1 \\
I\dot{\omega}_2 - I\omega_3\dot{\theta} &= G_2 \\
I\dot{\omega}_3 + I\omega_2\dot{\theta} &= G_3
\end{aligned}
$$

Where I is the ball moment of inertia, $\vec{\omega}$ is the angular velocity and $\vec{G}$ is the applied moment vector.

If, for instance, the applied moment is zero, then ω_1 will be constant and ω_2 and ω_3 will be governed by

$$
\begin{aligned}
\dot{\omega}_2 - \omega_3\dot{\theta} &= 0 \\
\dot{\omega}_3 + \omega_2\dot{\theta} &= 0
\end{aligned}
$$

or $\ddot{\omega}_3 + \omega_3\dot{\theta}^2 = 0$ assuming $\dot{\theta}$ to be constant and also $\ddot{\omega}_2 + \omega_2\dot{\theta}^2 = 0$.

Thus both ω_2 and ω_3 will have a cyclic variation with a frequency of $\dot{\theta}$. The nature of the applied moment $\vec{G}$ and its dependeance on ball angular velocity will of course greatly influence the ultimate motion of the balls.

As will be discussed in the next section, a knowledge of all of the above characteristic motions is necessary if any constraints on the ball motion are considered. In particular the knowledge of differences between the various components will be very useful and two classes of problems may be considered:

Class I: All frequencies are of the same general order

Class II: There is a distinct difference between the "very low" and "very high" frequency components.

Class I will be relevant for a relatively large bearing operating at very high speeds and Class II will apply to most small to moderate size bearings in a wide operating environment and large bearings operating at very low speeds, such as the DMA bearings. For the Class I system a generalized motion of each element must be determined while certain constraints can be imposed on Class II systems depending on the frequency range of interest. Also, some constraints may apply to class I system if the amplitudes of motion of certain components are very small compared to other components. A more extensive discussion of this subject is presented in the next section in terms of the modifications to the computer program DREB.

3.0 DREB MODIFICATIONS AND RAPIDREB DEVELOPMENTS

The available Dynamics of Rolling Element Bearings (DREB) computer program has been enhanced for several added capabilities for ball bearing performance simulation. The roller bearing capacilities of DREB have been suppressed in this enhanced version named RAPIDREB. All program modifications are centered around speeding up the computation and examining the low frequency behavior in the generalized performance simulation. Hence the modifications fall in two catagories:

1. Suppression of High Frequency Content
2. Improved integrating algorithms.

Aside from the above, a minor modification to extend the program capabilities to treat ball guided cages for DMA bearings has also been made. This section of the report briefly discusses all these program enhancements.

3.1 Suppression of High Frequency Content

As discussed in the preceding section there are certain high frequency characteristics present in the general performance simulation of ball bearings. A careful suppression of these frequencies is necessary to efficiently simulate the low frequency phenomenon specially because the time step size in the numerical integration algorithm is determined by the highest frequency present in the system. DREB is modified in two ways to suppress the high frequency motion, e. g., ball motion constraints and suitable damping.

3.1.1 Ball Motion Constraints

The very high frequency motion results from ball vibration between the two races. Both the elastic contact frequency and the kinematic frequency represent the motion of ball mass center relative to the supporting races, as discussed in the preceding section. If the applied loads on the bearing are free of any high frequency vibrational loads, the amplitudes of the ball vibration are generally small and the resulting changes in contact loads are fairly insignificant when compared with the nominal loads. Thus it may be reasonable to constrain the ball such that the ball center follows a definite path which satisfies equilibrium of all normal contact forces acting on the ball including the centrifugal forces. The traction forces, being quite small compared to the normal forces, may be neglected in the equilibrium equations. Also an equilibrium of forces is carried out on the inner race and its motion is constrained to satisfy such an equilibrium. Such constraints are reasonable for all static applied loads and constant races speeds. They are also applicable to time varying loads and speeds provided the frequency of such variations is very small compared to the characteristic ball/race vibration frequencies. Thus moderate race accelerations and synchronous loading due to unbalance can be satisfactorily treated within the realm of such equilibrium constraints.

It is true that the process of imposing the above constraints is similar to obtaining the conventional quasi-static solutions at each time step and hence a set of non-linear algebraic equations has to be solved. This may appear to be quite time consuming but even for relatively large steps, allowable by ball/race traction phenomenon, the changes in the equilibrium conditions are rather small and therefore the convergence of the equilibrium equations is very rapid. Thus in spite of the additional effort of solving the equilibrium equations at each step, the equilibrium constraints result in a substantial reduction in the overall computational effort, primarily due to the relatively large time step size permissible by suppressing the very high frequency motion.

3.1.2 Damping Considerations

There may be some damping considerations which can assist in eliminating certain high frequencies. It is true that actual damping at any contact interface in the bearing is quite small and therefore the introduction of any damping will be "fictitious" with the prime objective of eliminating the high frequency response without significantly altering the very low frequency behavior.

Any contact interface can be modeled in terms of a spring with stiffness k (N/M) and a dashpot with damping C (NS/M) as shown in Figure 3-1. The contact stiffness will be known from the Hertzian contact parameters at the particular contacts and the damping coefficient will be the required input. Perhaps specification of damping will be best in terms of a damping ratio C/C_c, where C_c is called critical damping [7] corresponding to the

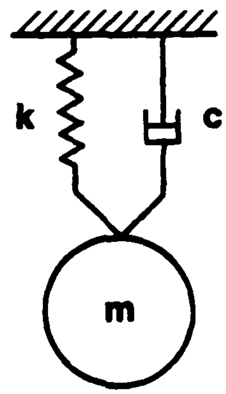

Fig. 3-1 Simple Spring and Damper Model for any Contact Interface

undamped natural frequency of the system.

Damping ratio $\zeta = C/C_c$

Where $C_c = 2m\omega_n$

$$\omega_n = \sqrt{\frac{k}{m}}$$

Thus ζ can be specified for the ball/race, ball/cage and cage/race interfaces. For the purpose of "real" damping ζ will be very small for most steels but for certain nonmetals used for fabricating the cages it may be appreciable. In any event any non-zero value of ζ will only effect the general response in the neighborhood of the undamped natural frequency ω_n [7] and the response at very low frequencies will be practically unchanged.

Each contact interface in the bearing is modeled in terms of the above simplified vibration model and the computer program DREB is modified accordingly.

3.2 Integrating Algorithms

Algorithms used to integrate a system of first order differential equations may be either of implicit or explicit type. The implicit algorithms are better known as the predictor corrector formulae and such schemes can be very efficient if no discontinuities and high frequency oscillatory motions are present. These methods make use of a set of existing solutions to extrapolate to an expected solution at the current time step and this is called the prediction step. With the predicted values the derivatives are computed and corrections to the predicted values are applied in an iterative manner. Normally two to three correction iterations are enough for most non-stiff systems. If no high frequency motion exists in the bearing and if there are no discontinuities in the solutions the predictor-corrector process can provide a relatively large time step and therefore speed up the computation in the desired time domain. However, if discontinuities are present, such as ball/cage or race/cage collisions, the extrapolation process may not be realistic and the integration process as a whole will exhibit stability problems. For such cases an explicit formula, better known as the Runga-Kutta method, which makes use of only one solution to march to the next step will probably be the best choice.

DREB is modified to incorporate both implicit and explicit methods, with appropriate step changing criterion. The step changing process generally depends on the local truncation error at each step. An estimate of this error is quite straight forward for an implicit method but it may be quite difficult for an explicit method. On the other hand treatment of variable size steps is trivial for an explicit scheme while considerable complications arise in the extrapolation process of the predictor. A special predictor-corrector formula in terms of a divided difference notation is therefore developed to efficiently carry out the extrapolation. Some suitable explicit Runga-Kutta methods are then reviewed with particular emphasis on error estimates and step changing rationale.

3.2.1 Adams Type Predictor-Corrector in Divided Differences

A predictor-corrector algorithm for a differential equation with one dependent variable is developed in this section. The extension of the algorithm to a system is quite obvious. Let the governing differential equation be written as:

$$\frac{\partial y}{\partial x} = f(x,y), \qquad (1)$$

with initial condition $y_o = y(x_o)$.

It will be further assumed that solutions in the domain $0 \leq x \leq x_n$ have been obtained by some explicit algorithm and the predictor-corrector formula, to be developed here, will advance the solution from x_n to x_{n+1}. The process can be written as:

$$y_{n+1} = y_n + \int_{x_n}^{x_{n+1}} f(x,y)\, dx \qquad (2)$$

With certain polynomial approximations to f(x,y), the required algorithm can be developed. The formulation presented below is very similar to the conventional Adams-Bashford-Moulton formulae.

3.2.1.1 Predictor Formula

Let the solutions (y_n, y_{n-1}, ... y_{n-N}) and the derivatives (f_n, f_{n-1}, ..f_{n-N}) be available at the (N+1) points (x_n, x_{n-1}, ... x_{n-N}) and for any x, let the derivative $f(x)^*$ be a Nth order polynomial passing through the points $(x_n, x_{n-1}, \ldots x_{n-N})$, i.e.:

$$f(x) = f(x_n) + \sum_{i=1}^{N} (x-x_n)(x-x_{n-1}) \cdots (x-x_{n-i+1})\, f[x_n, x_{n-1} \cdots x_{n-i}]$$
$$+ (x-x_n)(x-x_{n-1}) \cdots (x-x_{n-N})\, f[x_n, x_{n-1} \cdots x_{n-N}, \xi] \qquad (3)$$

Where $x_{n-N} \leq \xi \leq x_{n+1}$ and the conventional divided difference notion [8] is used. The divided difference notation will be convenient for step size change and will eliminate the need for interpolation to points spaced uniformly with a constant grid spacing.

The predictor formula is obtained by substituting the polynomial approximation, equation (3), in equation (2). Thus

$$y_{n+1} = y_n + \sum_{i=0}^{N} f[x_n, x_{n-1} \cdots x_{n-i}] \sum_{j=0}^{i} \frac{x_{n+1}^{j+1} - x_n^{j+1}}{(j+1)} \cdot A_j$$
$$+ f[x_n, x_{n-1} \cdots x_{n-N}, \xi] \sum_{j=0}^{N+1} \frac{x_{n+1}^{j+1} - x_n^{j+1}}{(j+1)} \cdot A_j \qquad (4)$$

Where A_j are the coefficients of the polynomial

$$(x-x_n)(x-x_{n-1}) \cdots (x-x_{n-i}) \equiv A_o + A_1 x + A_2 x^2 + \cdots A_{i+1} x^{i+1}$$

Note that estimate of ξ will be required in order to estimate the trunction error term of equation (4). It will be reasonable to assume $\xi = x_{n+1}$ and the corresponding solution as the predicted value.

From computational standpoint, it may be noted that as the order N increases, the term multiplying the polynomial coefficient A_j may experience a round off problem, which can be eliminated by scaling the independent variable as

* Note the change in notation: the derivative is actually a function of y also, notation used here is f(x) = f(x,y).

$$S \equiv \frac{x - x_p}{x_{p+1} - x_p} = \frac{x - x_p}{\Delta_p} \tag{5}$$

so that the polynomial

$$(x-x_n)\ (x-x_{n-1}) \ldots . (x-x_{n-j}) \equiv \Delta_p^{j+1}\ (S-x_n^*)\ (S-x_{n-1}^*) \ldots . (s-x_{n-j}^*)$$

where $x_j^* = (x_j - x_p)/\Delta_p$

Also if $p=n$, the integral of equation (2) takes the convenient form:

$$\int_{x_n}^{x_{n+1}} \{(x-x_n) \ldots . (x-x_{n-j})\}\ dx = \Delta_n^{j+2} \int_0^1 \{(S-x_n^*)\ (S-x_{n-1}^*) \ldots .. (S-x_{n-j}^*)\}\ dS$$

$$= \Delta_n^{j+2} \sum_{k=0}^{j+1} \frac{A_k^*}{k+1} \tag{6}$$

Where A_k^* are coefficients of the polynomial

$$(S-x_n^*) \ldots .. (S-x_{n-j}^*) \equiv A_0^* + A_1^* S + \ldots .. A_{n-j+1}^* S^{n-j+1}$$

Substituting (5) and (6) in equation (4) gives the final predictor formula

$$y_{n+1} = y_n + \sum_{i=0}^{N} f[x_n, x_{n-1}, \ldots .\ x_{n-i}]\ \Delta_n^{i+1} \sum_{j=0}^{i} \frac{A_j^*}{j+1}$$

$$+ f[x_n, x_{n-1}, \quad x_{n-N}, x_{n+1}]\ \Delta_n^{N+2} \sum_{j=0}^{N+1} \frac{A_j^*}{j+1} \tag{7}$$

It can easily be seen from equation (5) that the coefficients A_j^* will only have to be computed if the grid in x is non-uniform.

3.2.1.2 Corector Formula

For the general corrector formula, let f(x) be a polynomial of degree M, passing through the (M+1) points $(x_{n+1}, x_n, x_{n-1} \ldots . x_{n-M+1})$. Again using the divided difference notations

$$f(x) = f(x_{n+1}) + \sum_{i=0}^{M-1} (x-x_{n+1})\ (x-x_n)\ (x-x_{n-1}) \ldots . (x-x_{n-i+1})\ f[x_{n+1}, x_n, x_{n-1}, \ldots \ldots . x_{n-i}]$$

$$+ (x-x_{n+1})\ (x-x_n) \ldots . (x-x_{n-M+1})\ f[x_{n+1}, x_n \ldots . x_{n-M+1}, \xi] \tag{8}$$

Substitution of (8) in equation (2) gives the required corrector formula.

$$y_{n+1} = y_n + \sum_{i=-1}^{M-1} f[x_{n+1},x_n,\ldots x_{n-i}] \sum_{j=0}^{i+1} \frac{x_{n+1}^{j+1}-x_n^{j+1}}{(j+1)} B_j$$

$$+ f[x_{n+1},x_n,\ldots x_{n-M+1}, \xi] \sum_{j=0}^{M+1} \frac{x_{n+1}^{j+1} - x_n^{j+1}}{(j+1)} B_j \qquad (9)$$

Where B_j are coefficients of the polynomial

$$(x-x_{n+1})\ (x-x_n)\ (x_{n-1})\ \ldots(x-x_{n-i}) \equiv B_0 + B_1\ x + \ldots\ B_{i+2}\ x^{i+2}$$

Again an estimate of the truncation error will require the value of ξ which may be assumed as x_{n-M}, which demands that one additional data point is available. This demand is easily met if $M \leq N$.

Scaling the independent variable again, as done for the predictor formula will give the final corrector algorithm:

$$y_{n+1} = y_n + \sum_{i=-1}^{M-1} f[x_{n+1},x_n,\ldots x_{n-i}]\ \Delta_n^{i+2} \sum_{j=0}^{i+1} \frac{B_j^*}{j+1}$$

$$+ f[x_{n+1},x_n\ldots x_{n-M+1},x_{n-M}]\ \Delta_n^{M+2} \sum_{j=0}^{M+1} \frac{B_j^*}{j+1} \qquad (10)$$

As for the predictor formula, the coefficients B_j^* will only have to be computed for non-uniform grid. Also it can be easily seen that the truncation error will employ the last term in the divided difference table and it will be proportional to Δ_n^{M+2}.

3.2.1.3 Computation of Coefficients

In order to implement equations (7) and (10) a formula for computing the coefficients of the polynomial

$$P \equiv (x-x_n)\ (x-x_{n-1})\ldots(x-x_{n-p})$$

will be required.

By expanding the polynomial for p = 1,2,3,4 and by noting the algebraic similarities in the expansions, the polynomial can be written in the following general form:

$$P \equiv x^{p+1} + \sum_{k=1}^{p+1} x^{p-k+1} (-1)^k \sum_{i_1=1}^{p-k+2} x_{n-i_1+1} \sum_{i_2=i_1+1}^{p-k+3} x_{n-i_2+1} \cdots \sum_{i_j=i_{j-1}+1}^{p-k+j+1} x_{n-i_j+1} \cdots$$

$$\cdots \sum_{i_k=i_{k-1}+1}^{p+1} x_{n-i_k+1} \tag{11}$$

The above formula can be coded once the maximum value of p is assigned.

3.2.1.4 Step Changing Criterion

The step size can be changed at any time in the above formulation by knowing the truncation error at the preceding step size and the allowable limit. Let us assume that the order of corrector is equal to that of the predictor (M=N) and hence the above algorithm is a (N+1) step process. The truncation error of the predictor-corrector step will now be given by the truncation error of the corrector step only [9]. Thus error committed in arriving at x_{n+1}, from x_n is given by:

$$\frac{E_{n+1}}{\Delta_n^{N+2}} = f[x_{n+1}, x_n \ldots x_{n-N}] \sum_{j=0}^{N+1} \frac{B_j^*}{j+1} \tag{12}$$

An estimate of step Δ_{n+1} in going from x_{n+1} to x_{n+2} can be obtained from the relation:

$$\Delta_{n+1} = \frac{\Delta_n}{C} \left(\frac{\varepsilon}{E_{n+1}} \right)^{\frac{1}{N+2}} \tag{13}$$

Where ε is the allowable truncation limit and C (> 1) is a constant, which arbitrarily reduces the computed step size with the expectation that the tolerance criterion will be met at x_{n+2}. The value of C may be selected primarily by experience, it may vary from 1.2 to 2.0. Also it should be noted that any time the step size is changed all coefficients will have to be computed for N+1 steps (assuming that no step change occurs within these steps) and hence this added work may outweigh the savings in increased step size. It is therefore recommended that the step size be changed only if the following conditions is met:

$$\frac{\Delta_{n+1}}{\Delta_n} > a \qquad (14)$$

Where a is a constant, again selected by experience; it may be assumed as 1.25 as a first approximation.

Equation (13) can also be used to reduce the step size at any time, if the truncation limit is not met.

From a practical standpoint, upper and lower limits to the step size are assigned and the objective of the step changing rationale is to keep the step size to a maximum within this range. Also if the truncation limit is not met for a specified number of steps then further integration should be terminated.

3.1.2.5 Change of Order

A criterion for change of order is not very clear and generally a trial and error approach is used. At a given time, solutions with the order reduced and increased by one are computed and the order giving minimum truncation is selected. Similar to the step size the maximum and minimum bounds on the order should also be assigned. The maximum order can generally be determined by examining the divided differences which will generally decrease with increase of order. However, the computed values may show an increasing trend at very high order, which will commonly be due to machine round off errors. Thus the maximum order should be limited by this phenomenon. Selection of minimum order will primarily be by experience with the problem being investigated.

3.1.2.6 Computational Considerations

In developing a computer code for implementing the above formulation, certain considerations will be helpful:

1. Set a maximum limit on the order and hence the number of solutions to be stored.
2. Compute divided differences of derivatives at the first call and upgrade table at subsequent calls by adding the latest solution and deleting the earliest solution.

3. Check grid uniformity and compute coefficients (in subsequent calls) only in case of non-uniformity resulting from step size change.

4. Check maximum allowable order by examining the divided difference table and make sure that the current order is less than this maximum.

5. Store the entire divided difference table in disc file at the end of the run for subsequent restarts of the program.

3.2.2 Explicit Runga-Kutta Type Algorithms

As discussed earlier an explicit formula is required for two purposes. First for integrating the equations in the domain of time where there are either discontinuities or large cyclic variations at relatively high frequency. Secondly an explicit formula is required to start the implicit predictor-corrector process. Also it is required that some estimate of the local truncation error be available in order to establish some criterion for changing the step size.

The existing DREB program employs a Runga-Kutta-Merson method [9] which does estimate the truncation error. It is a fourth order process and requires an additional function evaluation for estimating the error. Hence a total of five function evaluations per step are used. There have been some questions about the accuracy of the error estimate provided by this method particularly when the equations are non-linear. Hence methods due to Scranton and England have appeared in the literature [10]. The Scranton method also uses five function evaluation but the error estimate is only realistic for a single equation and its validity to a system of equations is questionable. The England method attempts to resolve this problem by an additional function evaluation per step and it is therefore a six stage process. Mcre recnetly Fehlberg [11] has provided another six stage formula which has been of considerable use in recent years. Since the selection of a method really depends on the problem under consideration, which in the case of a ball bearing can be quite complicated by the operating conditions, both the formulae due to England and Fehlberg are incorporated in the RAPIDREB version of DREB.

All of the explicit formulae can be best summarized in terms of tabular notation [12]. Thus with given solution y_n at the nth step, the solution y_{n+1} at (n+1)th step is given by

$$y_{n+1} = y_n + \sum_{i=1}^{n} \gamma_i k_i \tag{15}$$

where

$$k_i = h\, f(y_n + \sum_{j=1}^{i-1} \beta_{ij} k_j) \quad i = 1,\ldots.n \tag{16}$$

and the estimate of local truncation error E_{n+1} is given by

$$E_{n+1} = \sum_{i=1}^{n} \gamma_i^* k_i \tag{17}$$

The values of β, γ and γ^* for the three methods discussed above are given in Tables 3.1 to 3.3.

All of the above methods are fourth order. For improved accuracy a higher order method may sometimes be desirable, e.g., in a detailed simulation of one ball/cage collision. With this objective a fifth order Kutta-Nystrom method [10] and a sixth order (eight stage) Huta method [10] are also incorporated in the computer program. The coefficients of these methods are given respectively in Tables 3.4 and 3.5. For these methods since γ^* is not available a step doubling technique is used to compute the truncation error. In other words the computation at each step is done twice, first with the prescribed step size and then with half the prescribed step size. Thus the computation with these methods can be very expensive but the solutions obtained will be extremely accurate. Just for reference purposes the classical fourth order Runga-Kutta method, the fourth order Ralston-Runga-Kutta method, and the third and second order Runga-Kutta methods are also incorporated in the computer program. Again a step doubling technique is used in all these methods and hence considerable care must be exercised in their use. The coefficients for all these methods are fairly well known and they are summarized in Tables 3.6 to 3.9 for completeness.

TABLE 3.1

COEFFICIENTS FOR THE RUNGA-KUTTA-MERSON METHOD

i	β_{ij}				γ_i	γ_i^*
1					$\frac{1}{6}$	$-\frac{1}{3}$
2	$\frac{1}{3}$				0	0
3	$\frac{1}{6}$	$\frac{1}{6}$			0	$-\frac{3}{2}$
4	$\frac{1}{8}$	0	$\frac{3}{8}$		$\frac{2}{3}$	$-\frac{4}{3}$
5	$\frac{1}{2}$	0	$-\frac{3}{2}$	2	$\frac{1}{6}$	$\frac{1}{6}$

TABLE 3.2

COEFFICIENTS FOR THE RUNGA-KUTTA-ENGLAND METHOD
$n = 6$

i	β_{ij}					γ_i	γ_i^*
1						$\frac{1}{6}$	$-\frac{1}{8}$
2	$\frac{1}{2}$					0	0
3	$\frac{1}{4}$	$\frac{1}{4}$				$\frac{2}{3}$	$-\frac{2}{3}$
4	0	-1	2			$\frac{1}{6}$	$-\frac{1}{16}$
5	$\frac{7}{27}$	$-\frac{10}{27}$	0	$\frac{1}{27}$		0	$\frac{27}{56}$
6	$\frac{28}{625}$	$-\frac{1}{5}$	$\frac{546}{625}$	$\frac{54}{625}$	$-\frac{378}{625}$	0	$\frac{125}{336}$

TABLE 3.3

COEFFICIENTS FOR THE RUNGA-KUTTA-FEHLBERG METHOD

$n = 6$

i	β_{ij}					γ_i	γ_i^*
1						$\frac{16}{135}$	$\frac{1}{360}$
2	$\frac{1}{4}$					0	0
3	$\frac{3}{32}$	$\frac{9}{32}$				$\frac{6656}{12825}$	$\frac{196992}{6579225}$
4	$\frac{1932}{2197}$	$-\frac{7200}{2197}$	$\frac{7296}{2197}$			$\frac{28561}{56430}$	$\frac{41743}{1429560}$
5	$\frac{439}{216}$	-8	$\frac{3680}{513}$	$-\frac{845}{4104}$		$-\frac{9}{50}$	$\frac{1}{50}$
6	$-\frac{8}{27}$	2	$-\frac{3544}{2565}$	$\frac{1859}{4104}$	$-\frac{11}{40}$	$\frac{2}{55}$	$\frac{2}{55}$

TABLE 3.4

COEFFICENTS FOR THE FIFTH ORDER KUTTA-NYSTROM METHOD

$n = 6$

i	β_{ij}					γ_i
1						$\frac{23}{192}$
2	$\frac{1}{3}$					0
3	$\frac{4}{25}$	$\frac{6}{25}$				$\frac{125}{192}$
4	$\frac{1}{4}$	-3	$\frac{15}{4}$			0
5	$\frac{2}{27}$	$\frac{10}{9}$	$-\frac{50}{81}$	$\frac{8}{81}$		$-\frac{27}{64}$
6	$\frac{2}{25}$	$\frac{12}{25}$	$\frac{2}{15}$	$\frac{8}{75}$	0	$\frac{125}{192}$

TABLE 3.5

COEFFICIENTS FOR THE SIXTH ORDER (EIGHT STAGE) HUTTA METHOD

$n = 8$

i	β_{ij}							γ_i
1								$\frac{41}{840}$
2	$\frac{1}{9}$							0
3	$\frac{1}{24}$	$\frac{1}{8}$						$\frac{9}{35}$
4	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$					$\frac{9}{280}$
5	$-\frac{5}{8}$	$\frac{27}{8}$	-3	$\frac{3}{4}$				$\frac{34}{105}$
6	$\frac{221}{9}$	-109	$\frac{289}{3}$	$-\frac{34}{3}$	$\frac{1}{9}$			$\frac{9}{280}$
7	$-\frac{183}{48}$	$\frac{113}{8}$	$-\frac{53}{6}$	$-\frac{11}{8}$	$\frac{10}{6}$	$\frac{1}{16}$		$\frac{9}{35}$
8	$\frac{358}{41}$	$-\frac{2079}{82}$	$\frac{501}{41}$	$\frac{417}{41}$	$-\frac{227}{41}$	$-\frac{9}{82}$	$\frac{36}{41}$	$\frac{41}{840}$

TABLE 3.6

COEFFICIENTS OF THE CLASSICAL FOURTH ORDER RUNGA-KUTTA METHOD

$n = 4$

i	β_{ij}			γ_i
1				$\frac{1}{6}$
2	$\frac{1}{2}$			$\frac{1}{3}$
3	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{3}$
4	1	0	1	$\frac{1}{6}$

TABLE 3.7

COEFFICIENTS OF THE FOURTH ORDER RALSTON-RUNGA-KUTTA METHOD

n = 4

i	β_{ij}			γ_i
1				0.17476028
2	0.40			- 0.55148053
3	0.29697760	0.15875966		1.20553547
4	0.21810038	- 3.05096470	3.83286432	0.17118478

TABLE 3.8

COEFFICIENTS OF THE THIRD ORDER RUNGA-KUTTA METHOD

n = 3

i	β_{ij}		γ_i
1			$\frac{1}{4}$
2	$\frac{1}{3}$		0
3	0	$\frac{2}{3}$	$\frac{3}{4}$

TABLE 3.9

COEFFICIENTS OF THE SECOND ORDER RUNGA-KUTTA METHOD

$n = 2$

i	β_{ij}	γ_i
1		$\frac{1}{2}$
2	1	$\frac{1}{2}$

3.3 Modifications for Ball/Cage Guidance in DMA Bearings

An analytical model for the ball guided cages has been presented earlier [13]. The model assumes a conical end on a cylindrical pocket as shown in Figure 3-2. It is really a purely geometric interaction model in the sense that the ball center is located with respect to the center of the cone base. This is shown as vector $\vec{r}_{bp}^{\,p}$ in the pocket frame in Figure 3-2.

In order to determine the magnitude of interaction with the conical surface, it will be convenient to transform $\vec{r}_{bp}^{\,p}$ into cylindrical coordinates (r, θ, z) such that

$$
\begin{aligned}
r &= \sqrt{(\tilde{r}_{bp_1}^{\,p})^2 + r_{bp_2}^{\,p})^2} \\
\theta &= \arctan\,(-\tilde{r}_{bp_1}^{\,p}/\tilde{r}_{bp_2}^{\,p}) \\
z &= \tilde{r}_{bp_3}^{\,p}
\end{aligned}
\tag{17}
$$

The angle θ measured as positive rotation about the z axis from the y axis will also denote the contact angle for the cage cone contact. It should be noted that if the cone exists on the inner diameter of cage the positive z axis will be pointing radially inwards and the y axis will also be rotated by 180°.

From simple geometry it may be shown that a unit vector along the normal to the conical surface from the center of the ball is given by

$$
\vec{e}_{bp}^{\,p} = \begin{Bmatrix} \cos\phi \, \sin\,(\theta + \theta') \\ \cos\phi \, \sin\,(\theta + \theta') \\ \sin\phi \end{Bmatrix} \tag{18}
$$

where ϕ is the cone angle and the angle θ is measured from the line denoting maximum interaction. Thus $\theta' = 0$, will correspond to the unit vector along maximum interaction.

The interaction δ_{bp} is given by

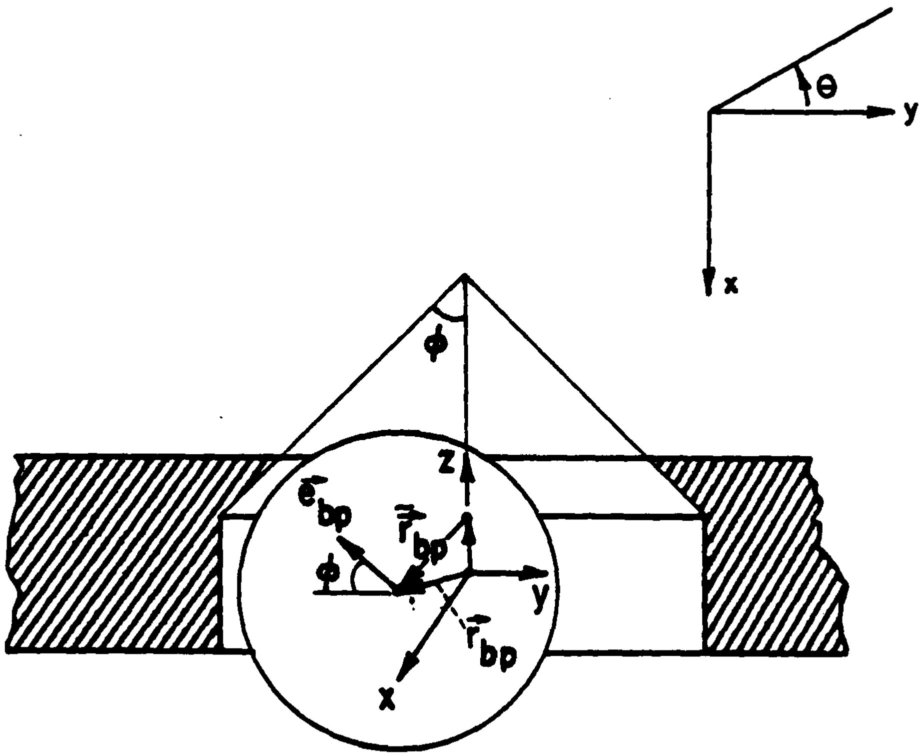

Fig. 3-2 **Exaggerated Pocket Geometry For a Ball Riding Cage**

812825

$$\delta_{bp}(\theta') = (\frac{d}{2} + \Delta_{bp}) \cos\phi - \vec{r}_{bp}^{\,p} \cdot \vec{e}_{bp}^{\,p} - \frac{d}{2} \tag{19}$$

It may be noted that since $\vec{e}_{bp}^{\,p}$ is a function of θ', the interaction will depend on $\theta'$and it will be necessary to determine the range of θ' for which δ_{bp} is negative. This range will also determine the length of interaction. Also by straightforward geometry, the relative position of the ball center and the length of the conical surface will determine if the vector $\vec{e}_{bp}$ falls on the given surface or not. The interaction analysis will only be performed when the vector $\vec{e}_{bp}$ falls on the given cage surface. Once the point of contact has been established the relative slip velocity can be readily defined in terms of the ball and cage velocities.

For the computed geometric interference, the normal load is approximated by either a Hertzian point or line contact, depending on the contact arc length. This is certainly an approximation because the contact between a sphere and a cone is not really Hertzian and the true solution will require a generalized three-dimensional elastic contact analysis. The tractive forces are computed by substituting the calculated slip velocities in the prescribed traction-slip relation. For the line contact the contact arc is divided into several incremental areas and an integration is performed to compute the normal and traction forces.

3.4 Recommendations for Using the RAPIDREB Version of DREB

All of the above enhancements are incorporated into the DREB program to create the new version RAPIDREB. In addition, with the objective of making the program efficient for ball bearings, the roller bearing parts and some special output options of DREB have been eliminated. Depending on the operational conditions, the computation effort in RAPIDREB may be significantly reduced if proper options are exercised. Some useful guidelines to this effect are therefore presented here:

1. If elastohydrodynamic lubrication exists it is recommended that a few time steps be run on the original DREB program and typical traction curves be printed out. Thus a ball to ball variation in traction

may be studied and a judgement can be made if the traction behavior can be approximated by a single curve and thereby eliminating all EHD computations at each step. If so, such a curve must be approximated for RAPIDREB.

2. For purely thrust loaded bearings the ball constraints can be set such that the load computation is performed only once. This will significantly effect the computational effort.

3. Bearings with a combined radial and thrust load may also employ ball/race constraints by performing an equilibrium analysis at each time step. This may result in an increase in the computational effort per step but the total effort will be greatly reduced by permitting the maximum step size to be much larger in absence of any high frequency motions.

4. Considerable care must be exercised in selecting the integration method. By extensive running of RAPIDREB it is found that the predictor-corrector scheme performs very well for cageless bearings with ball/race constraints. Hence this method is not recommended for most bearings with cage, where large discontinuities in the solutions may be present. The Fehlberg and England formulae do sometimes provide an increase in permissible step size but an increased function evaluation at every step adds to the effort per step. The benefit of these methods may only be investigated for prescribed application by a few trial runs. Any of the higher order methods provide extreme accuracy but they may be very time consuming and hence these methods should only be used in very small time domains.

5. For investigating the effects of small perturbations in the operating conditions, a steady state solution under nominal conditions must be obtained first and it should be used for determining the initial conditions for subsequent parametric runs.

6. Proper error processing control must be used to process time limit errors in order to store all the generated data files in the event an CP time limit is encountered. This is normally done by using the EXIT control statement on the CDC systems.

Appendix A contains the description of the input data to the RAPIDREB versions of both the main program RDREB and the plotting program RDREBP. The plot program is almost identical to the original version except for a few enhancements for handling large quantities of data and properly truncating the curves if they exceed the plot limits. This can be easily understood from the input description in Appendix A.

4.0 PERFORMANCE SIMULATION OF DMA BEARINGS

Ball bearings used in DMA (Despun Mechanical Assembly) systems are generally very large bearings operating at very low pure thrust loads and speeds. However, they are designed for a long time of continuous operation (of the order of ten years). The lubricant is sealed into the bearing at the time of assembly and hence lubricant degradation or starvation is not an uncommon problem. Also the nature of friction at ball/cage interface has been a suspect for cage instability in such applications.

Simulation of the dynamic performance of such bearings can be obtained for extended time domains by using the equilibrium constraints on ball motion in the RAPIDREB version of the computer program. This basically eliminates the ball/race natural vibration and thereby allows a substantially large time step to integrate the differential equations of motion over more than a shaft revolution. Such simulations are the subject of this section.

4.1 Bearing Geometry

Two different geometrical configurations of the DMA bearings are investigated. Figure 4-1 shows the geometrical details of a 100 mm bore angular contact bearing with the cage guided on the inner race and Figure 4-2 displays the design of a 150 mm bore ball bearing with the cage guided on the balls. The guidance surface is of a conical shape and it is attached to the cylindrical pockets on the inner diameter of the cage. Details of the cage geometry are shown in Figure 4-3.

4.2 Operating Conditions

Both bearings operate at relatively low pure thrust loads and speeds, which are typical of actual application. The operating condition assumed for the simulations to be presented here are:

100 mm bearing:	Thrust load	= 178 N
	Inner race speed	= 60 RPM
150 mm bearing:	Thrust load	= 534 N
	Inner race speed	= 62.5 RPM

```
RRRRRRRRRRRRRRR    DDDDDDDDDDDDDDD    RRRRRRRRRRRRRRR    EEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
RRRRRRRRRRRRRRRR   DDDDDDDDDDDDDDDD   RRRRRRRRRRRRRRRR   EEEEEEEEEEEEEEEE   BBBBBBBBBBBBBBBB
  RR         RR      DD        DD       RR         RR    EE                   BB         BB
  RR         RR      DD        DD       RR         RR    EE                   BB         BB
  RRRRRRRRRRRRR      DD        DD       RRRRRRRRRRRRR    EEEEEEEEEE           BBBBBBBBBBBBB
  RRRRRRRRRRRR       DD        DD       RRRRRRRRRRRR     EEEEEEEEEE           BBBBBBBBBBBBB
  RR      RR         DD        DD       RR      RR       EE                   BB         BB
  RR       RR        DD        DD       RR       RR      EE                   BB         BB
  RR        RR     DDDDDDDDDDDDDDDD     RR        RR     EEEEEEEEEEEEEEEE   BBBBBBBBBBBBBBBB
  RR         RR    DDDDDDDDDDDDDDD      RR         RR    EEEEEEEEEEEEEEEE   BBBBBBBBBBBBBBB
```

R A P I D   S I M U L A T I O N   O F   T H E

D Y N A M I C S   O F   R O L L I N G   E L E M E N T   B E A R I N G S

-A REAL TIME PERFORMANCE SIMULATION-
(VERSION RAPIDREB.0)

PRADEEP K. GUPTA
MECHANICAL TECHNOLOGY INCORPORATED
968 ALBANY-SHAKER ROAD
LATHAM, NEW YORK
U. S. A.

BEARING TYPE -- BALL   SPEC CODE -- 100MM-DMA-BRG IRG-CAGE 60RPM 178N**

BEARING GEOMETRY

BORE	(M) =	1.00000E-01	OUTER RACE SHRINK FIT	(M) =	1.00000E-05
OUTSIDE DIAMETER	(M) =	1.50000E-01	INNER RACE SHRINK FIT	(M) =	1.00000E-05
SHAFT INNER DIA	(M) =	8.00000E-02	HOUSING OUTER DIA	(M) =	2.00000E-01
BALL DIAMETER	(M) =	1.58750E-02	NUMBER OF BALLS	=	19
PITCH DIAMETER	(M) =	1.25000E-01	OUTER RACE CUR FACTOR	=	5.20000E-01
CONTACT ANGLE	(DEG) =	2.60000E+01	INNER RACE CUR FACTOR	=	5.20000E-01
END PLAY	(M) =	5.56731E-04	DIAMETRAL PLAY	(M) =	1.28532E-04
CAGE OUTER DIA	(M) =	1.35000E-01	CAGE OUTER DIA CLS	(M) =	3.50000E-03
CAGE INNER DIA	(M) =	1.15000E-01	CAGE INNER DIA CLS	(M) =	4.00000E-04
EFF CAGE WIDTH	(M) =	2.60000E-02	DIA RE/CAGE CLEARANCE	(M) =	3.00000E-04

	GUIDANCE TYPE	GUIDING RACE RADIUS (M)	GUIDING CAGE RADIUS (M)	EFF LAND WIDTH (M)	CAGE HALF WIDTH (M)
GUIDING LAND I	2	5.73000E-02	5.75000E-02	5.00000E-03	1.30000E-02
GUIDING LAND II	2	5.73000E-02	5.75000E-02	5.00000E-03	1.30000E-02

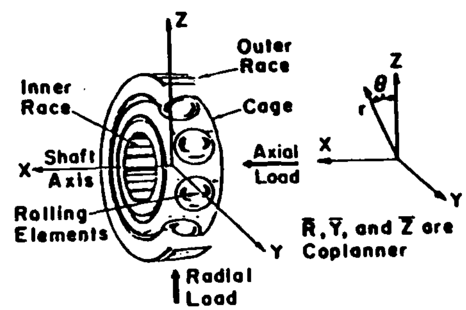

Fig. 4-1 Geometrical Data for the 100 mm DMA Ball Bearing with Inner Race Guided Cage

The outer race is assumed to be stationary in both cases and the operating temperature of both bearings is assumed to be 15°C. The motion of the cage is considered under the influence of gravity with the bearing axis oriented both along and normal to the direction of gravity. This is really the only factor which will promote cage motion and its interaction with the balls. The two orientations of the bearing with respect to the direction of gravity do represent laboratory conditions.

4.3 Ball/Race Traction Models

Traction behavior of typical lubricants used in DMA bearings is not fully known and a number of assumptions in modelling the behavior are generally made. For the present application a lubricant called Apiezon C + 1.5% antitoxidant [14, 15] is considered with the following properties under ambient pressure and at a reference temperature of 15°C.

Viscosity =	0.38425 Pa.S
Pressure-Viscosity Coefficient =	8.4234×10^{-9} 1/Pa
Temperature-Viscosity Coefficient =	6000°K
Thermal Conductivity =	0.10 N/S/°C

Since the traction parameters are not known, the following parameters, which actually correspond to a MIL-L-7808 type oil, are arbitrarily assumed for the Type I traction model discussed by Gupta et al [16].

Viscosity Parameter, μ^*	=	0.58889 Pa
Pressure-Viscosity Parameter, α^*	=	5.22136×10^{-9} 1/Pa
Temperature Viscosity Parameter, β^*	=	2.48550×10^{-2} 1/°K

Also typical of the DMA bearing environment a lubricant starvation factor (distance of lubricant maniscus from the edge of the Hertzian contact zone divided by the major contact half width) of 2 is assumed.

With the above lubricant parameters and the operating conditions outlined earlier the computed elastohydrodynamic traction coefficients for the two bearings are fairly linear with the slide-to-roll ratio, in the range of 0 to 0.10, as shown in Figure 4-4. The lubricant film thicknesses are

```
RRRRRRRRRRRRRRR     DDDDDDDDDDDDDDD     RRRRRRRRRRRRRRR     EEEEEEEEEEEEEEEE     BBBBBBBBBBBBBBB
RRRRRRRRRRRRRRRR    DDDDDDDDDDDDDDDD    RRRRRRRRRRRRRRRR    EEEEEEEEEEEEEEEE     BBBBBBBBBBBBBBBB
   RR         RR       DD         DD       RR         RR    EE                      BB         BB
   RR         RR       DD         DD       RR         RR    EE                      BB         BB
   RRRRRRRRRRRRR       DD         DD       RRRRRRRRRRRRR    EEEEEEEEEE              BBBBBBBBBBBB
   RRRRRRRRRRRR        DD         DD       RRRRRRRRRRRR     EEEEEEEEEE              BBBBBBBBBBBB
   RR      RR          DD         DD       RR      RR       EE                      BB         BB
   RR        RR        DD         DD       RR        RR     EE                      BB         BB
   RR          RR   DDDDDDDDDDDDDDDD       RR          RR   EEEEEEEEEEEEEEEE     BBBBBBBBBBBBBBBB
   RR           RR  DDDDDDDDDDDDDDD        RR           RR  EEEEEEEEEEEEEEEE     BBBBBBBBBBBBBBB
```

R A P I D   S I M U L A T I O N   O F   T H E

D Y N A M I C S   O F   R O L L I N G   E L E M E N T   B E A R I N G S

-A REAL TIME PERFORMANCE SIMULATION-
(VERSION RAPIDREB.0)

PRADEEP K. GUPTA
MECHANICAL TECHNOLOGY INCORPORATED
968 ALBANY-SHAKER ROAD
LATHAM, NEW YORK
U. S. A.

BEARING TYPE -- BALL   SPEC CODE -- 150MM-DMA-BRG BG-CAGE 62.5RPM S34N***

BEARING GEOMETRY

BORE	(M)	= 1.50000E-01	OUTER RACE SHRINK FIT	(M)	=	1.00000E-05
OUTSIDE DIAMETER	(M)	= 2.25000E-01	INNER RACE SHRINK FIT	(M)	=	1.00000E-05
SHAFT INNER DIA	(M)	= 1.37000E-01	HOUSING OUTER DIA	(M)	=	2.44000E-01
BALL DIAMETER	(M)	= 2.22250E-02	NUMBER OF BALLS		=	16
PITCH DIAMETER	(M)	= 1.87500E-01	OUTER RACE CUR FACTOR		=	5.20000E-01
CONTACT ANGLE	(DEG)	= 1.80000E+01	INNER RACE CUR FACTOR		=	5.30000E-01
END PLAY	(M)	= 6.86790E-04	DIAMETRAL PLAY	(M)	=	1.08777E-04
CAGE OUTER DIA	(M)	= 1.97104E-01	CAGE OUTER DIA CLS	(M)	=	3.98780E-03
CAGE INNER DIA	(M)	= 1.77165E-01	CAGE INNER DIA CLS	(M)	=	4.28600E-04
EFF CAGE WIDTH	(M)	= 3.35280E-02	DIA RE/CAGE CLEARANCE	(M)	=	4.06400E-04
RE/CAGE CONE ANG	(DEG)	= 3.75000E+01	RE/CAGE CONE HEIGHT	(M)	=	-3.18353E-03

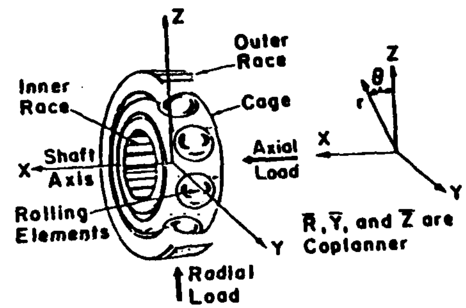

Fig. 4-2 Geometry of the 150 mm DMA Ball Bearing with Ball Guided Cage

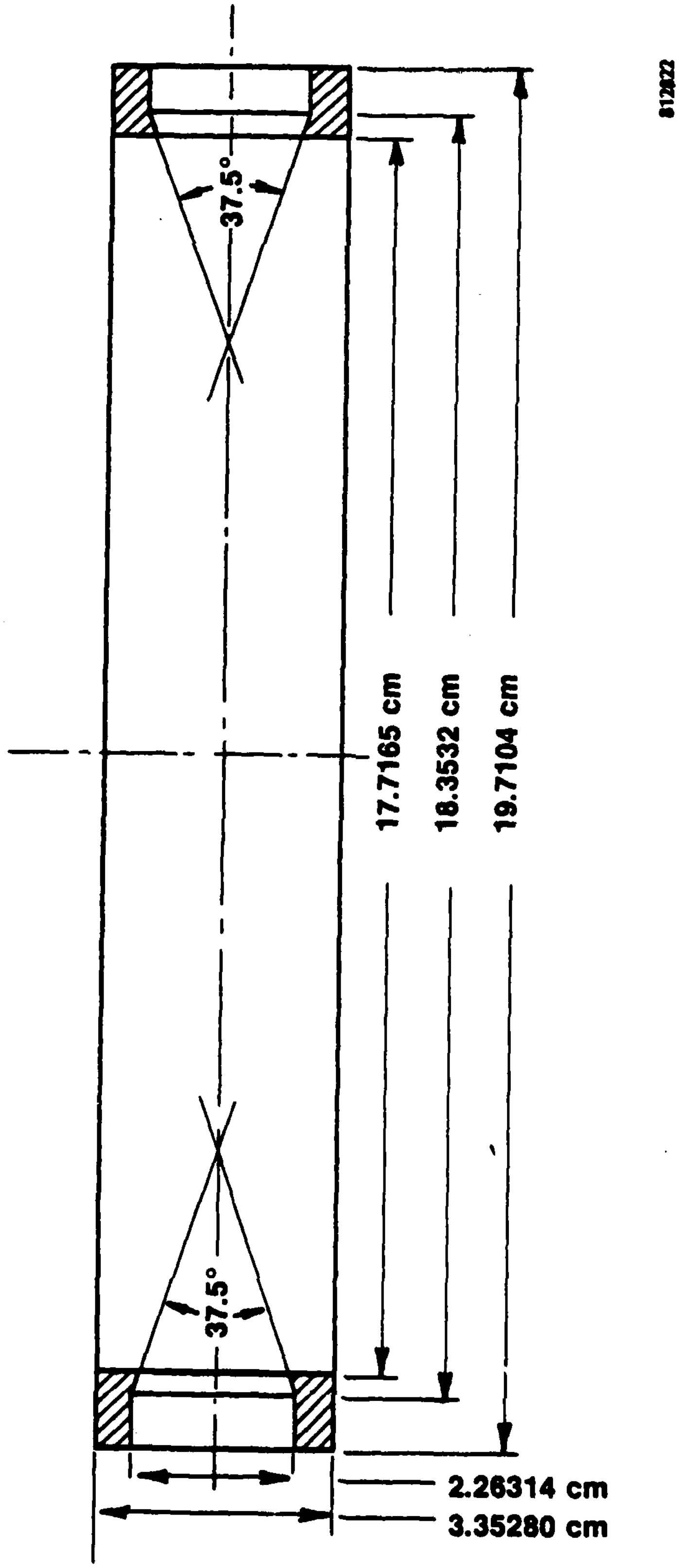

Fig. 4-3 Geometry of the Ball Guided Cage

also listed in the figure and from these numbers it may be concluded that the assumption of full film elastohydrodynamic lubrication even under starved conditions may be fairly reasonable. If the data of Figure 4-4 is plotted as a function of the sliding speed or slip velocity, then all of the data lies on one line with a slope of 0.0410 S/M; this is because the roll speed is different for each line in Figure 4-4. Thus a linear traction curve with a slope of 0.0410 S/M is assumed for both bearings.

Friction at ball/cage and cage/race contacts is another unknown and for the present investigation a constant friction coefficient of 0.010 is arbitrarily assumed.

4.4 Performance Simulations of 100 mm DMA Bearing

The first simulation with the RAPIDREB program is obtained for the 100 mm, inner race guided DMA ball bearing operating with gravity oriented along the bearing axis. In such a orientation the cage moves axially and rests on all the balls to balance its own weight. Typical ball/cage force variation is shown in Figure 4-5. The mean force is approximately equal to the cage weight. What is interesting to notice is the settling in of the ball/cage contact angle. The observed distinct frequency in this pattern is about 44.6 Hz and it is probably related to some kinematic phenomenon in the bearing; it is really much lower than the ball/cage contact frequency which is of the order of 1 kHz. Such a cyclic variation also gets transmitted to the cage angular acceleration about the bearing axis as shown in Figure 4-6, and hence to the skid parameter λ_2 as shown in Figure 4-7. The parameter λ_2 is defined as:

$$\lambda_2 = \frac{\text{Cage Angular Velocity}}{\text{Race Angular Velocity}}$$

From Figure 4-7 it is also seen that the cage does not experience any significant whirl. This is indicated by the parameter λ_1 in Figure 4-7 which is defined as:

$$\lambda_1 = \frac{\text{Cage Mass Center Whirl}}{\text{Cage Angular Velocity}}$$

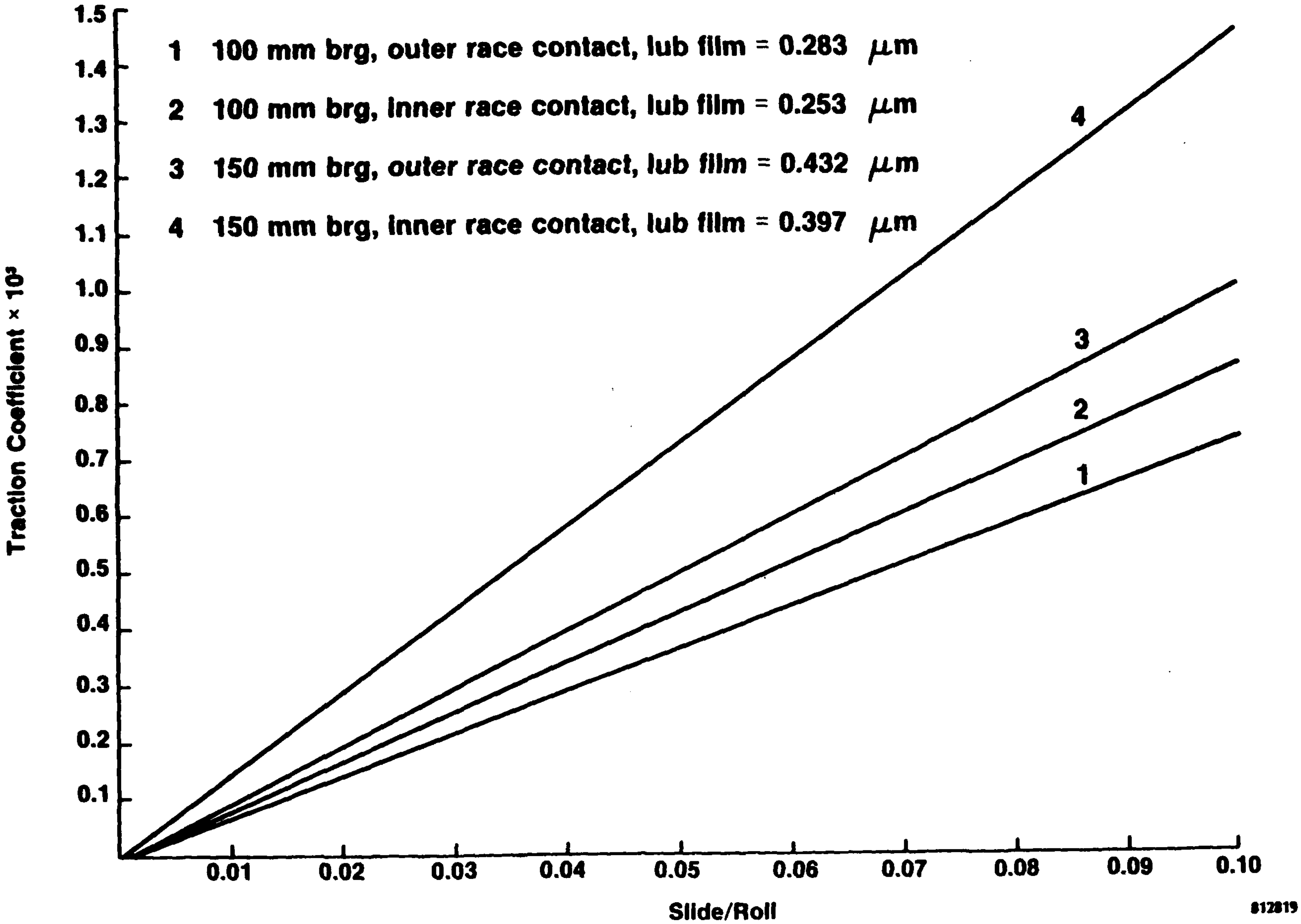

Fig. 4-4 Ball/Race Traction Characteristics for the DMA Bearings

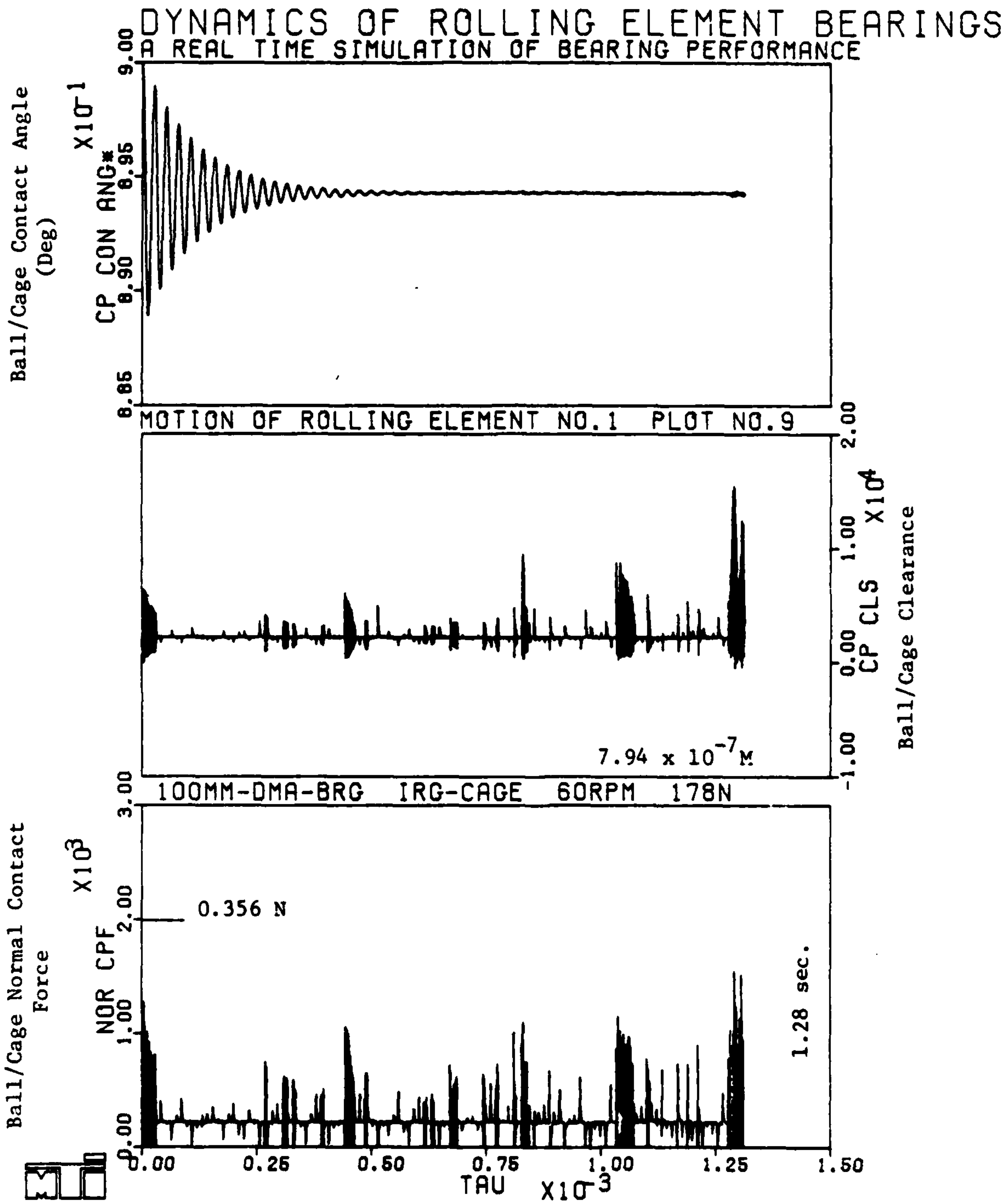

Fig. 4-5 Ball Cage Interation for the 100 mm DMA Ball Bearing

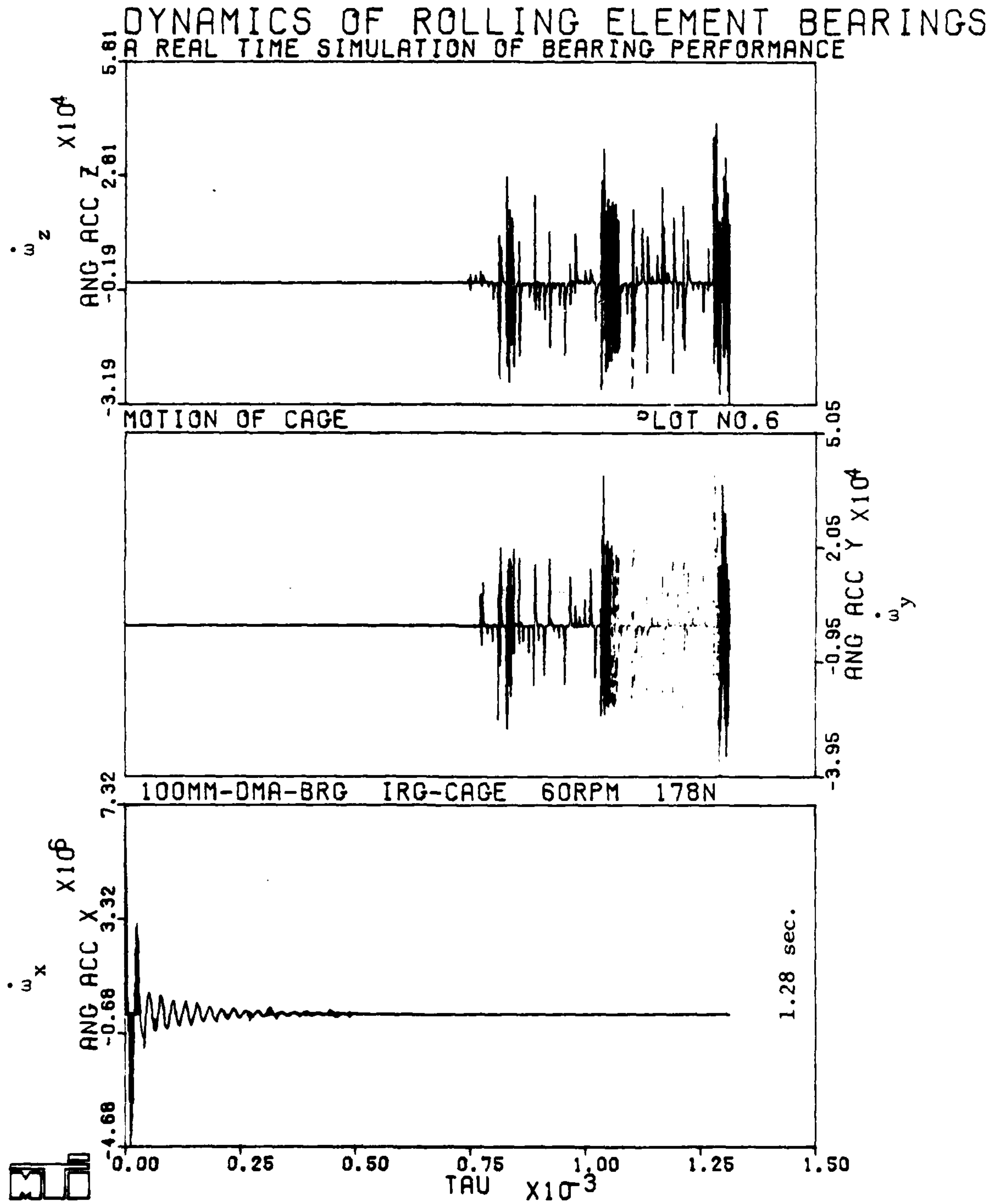

Fig. 4-6 Dimensionaless Cage Angular Accelerations for the 100 mm DMA Bearing with Gravity Acting Along the Bearing Axis. Scale = 1.319×10^7 RPM/S.

DYNAMICS OF ROLLING ELEMENT BEARINGS
A REAL TIME SIMULATION OF BEARING PERFORMANCE

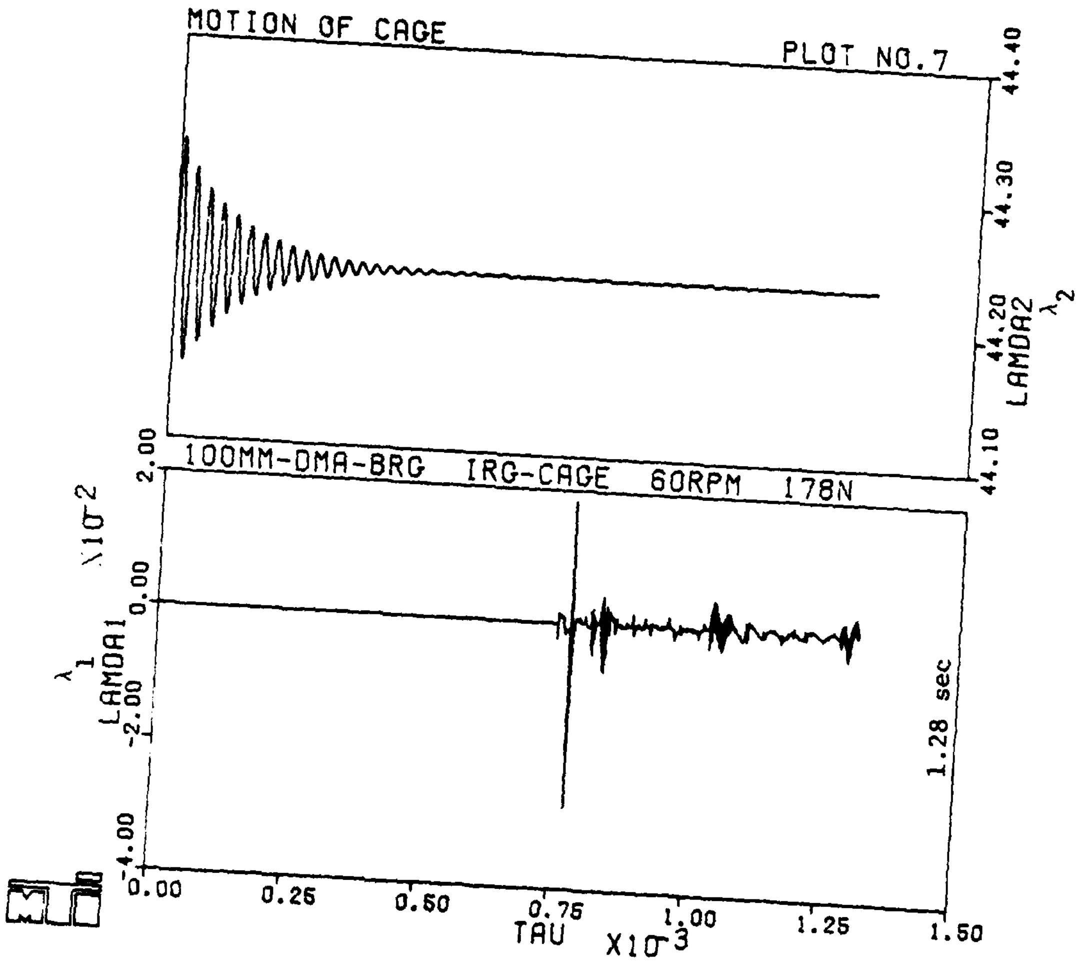

Fig. 4-7 Cage Whirl and Skid Parameters for the 100 mm DMA Bearings

The sudden change from zero whirl to some vibratory motion indicates cage radial motion beyond a certain machine round off threshold set in RAPIDREB. In reality the entire whirl pattern will look like what is seen in steady state in Figure 4-7, but the magnitude of the radial motion for all practical purposes will be negligible and no interaction with the guiding race will be seen. The bearing torque variations are seen in Figure 4-8. Again the cyclic variation corresponding to about 44.6 Hz is seen in both the outer and inner race torque. The damping of the cyclic variation is due to ball/race and ball/cage friction. Under actual laboratory conditions such cyclic torque variations may exist in steady state since the bearing is constantly subjected to subtle disturbances which may be adequate to continually excite the cyclic motion.

The ball motion indicates that the conventional race-control hypotheses do not hold when the ball/race friction is quite low as in the present case. As shown in Figure 4-9 the angular velocity vector tends to orient itself along the bearing axis and hence the x component continually increases while the z component decreases. Also a small component about the y axis is developed indicating gyroscopic slip. The resulting spin-to-roll ratios are shown in Figure 4-10. Such deviations of ball velocity from the race-control hypothesis have also been observed earlier [17].

When the bearing is oriented vertically such that the cage weight now acts in a plane perpendicular to the bearing axis, the cage tends to move radially and this causes substantial variations in ball/cage interactions as the ball travels around its orbit. Also significant interaction with the guiding race is observed and some coning motion (rotation about transverse axes of the cage) is also noted. Figure 4-11 shows the ball/cage force variation. The high frequency content is generally the ball/cage elastic contact frequency and the low frequency variation corresponds to the ball orbital velocity; the simulation time in Figure 4-11 corresponds to about one shaft revolution or approximately half of ball orbit. The cage/race forces are shown in Figure 4-12. The steady magnitude of the force is of the order of cage weight although not exactly equal to it since the ball/cage forces also enter in the cage equilibrium. Although not very clear

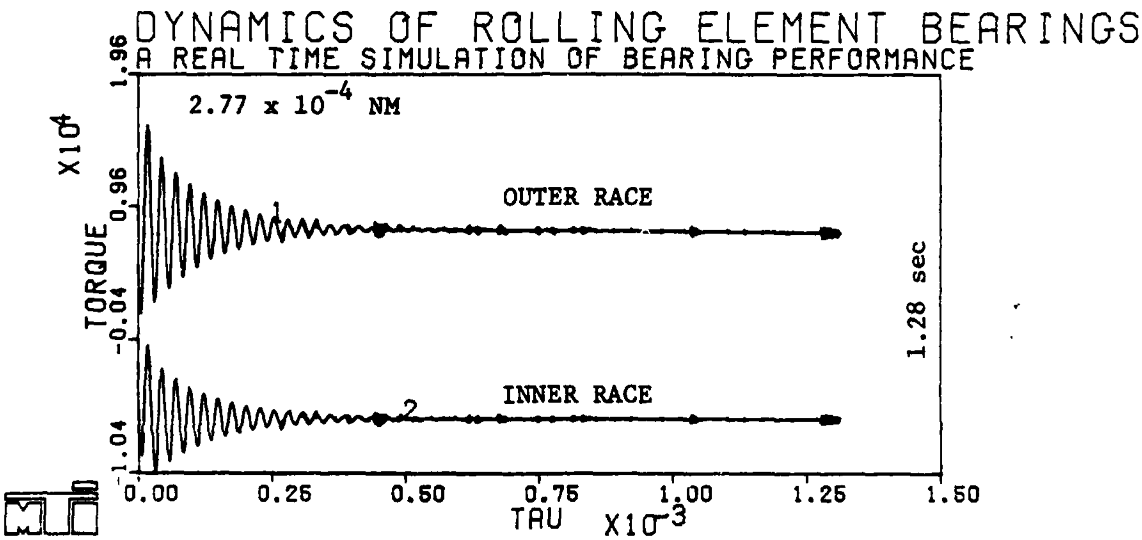

Fig. 4-8 Bearing Torque Variations for the 100 mm DMA Bearing with Gravity Acting Along the Bearing Axis

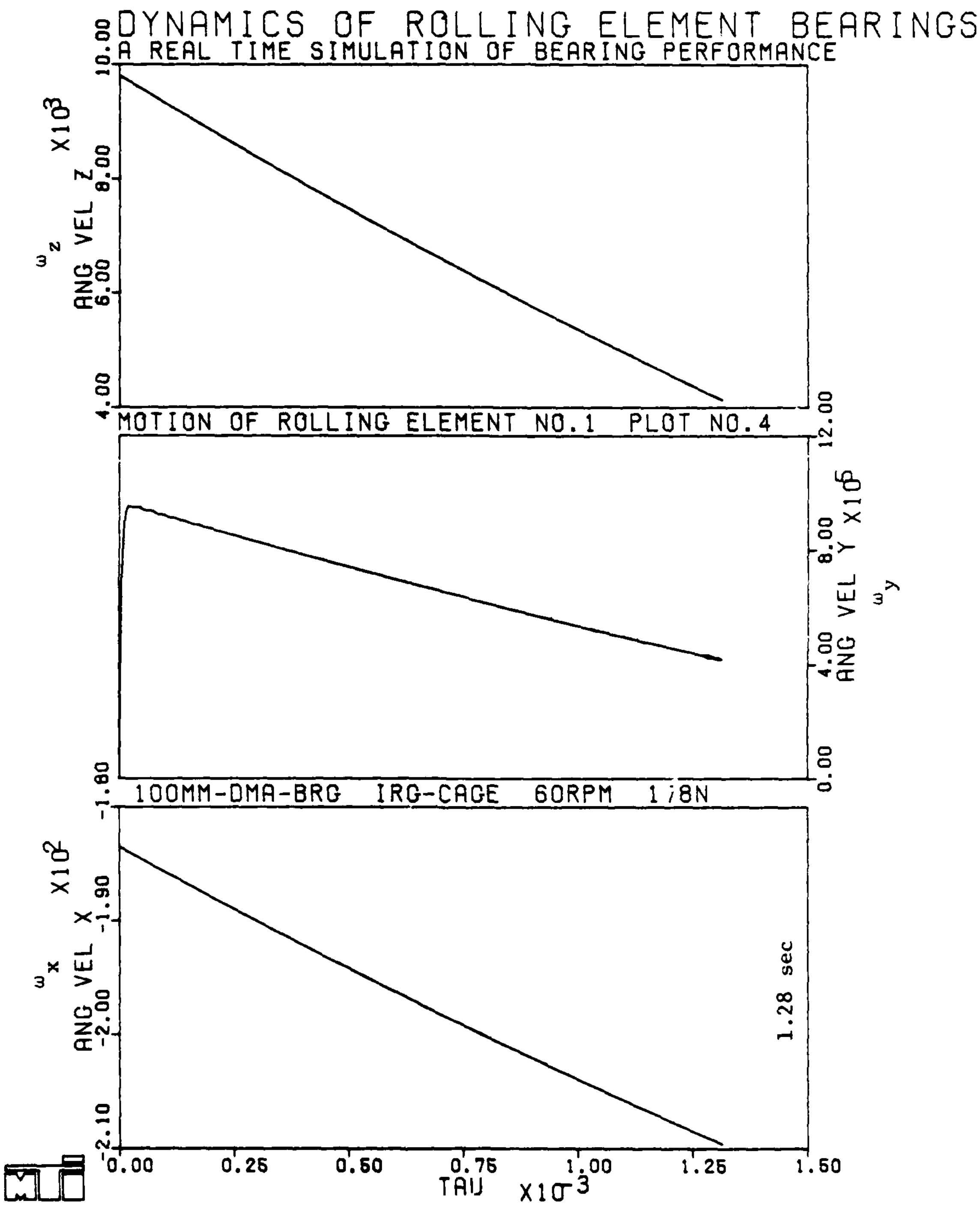

Fig. 4-9 Dimensionless Ball Angular Velocity for the 100 mm Bearing. Scale = 1.22×10^4 RPM

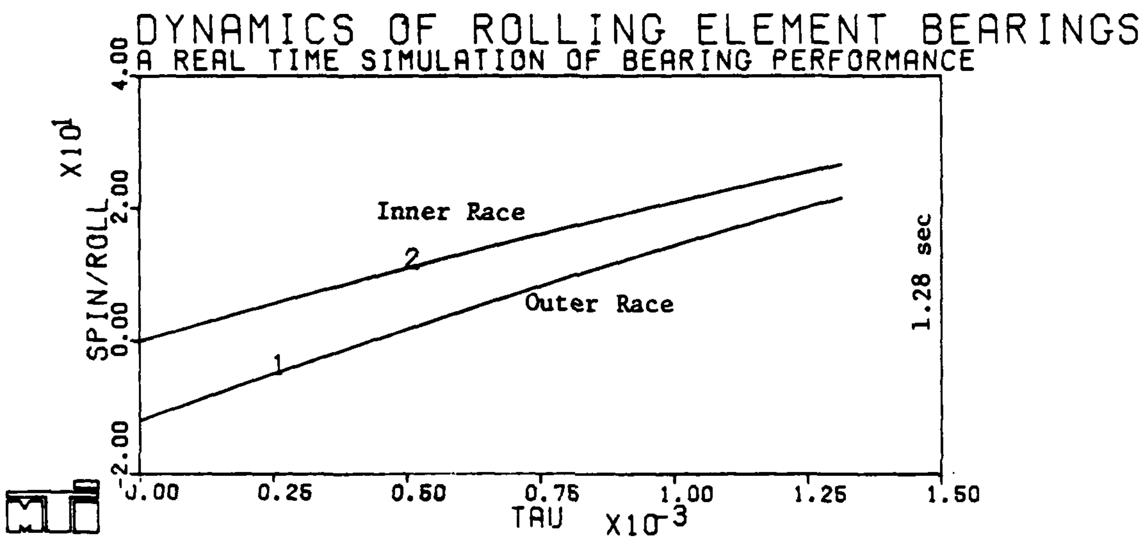

Fig. 4-10 Ball/Race Spin-to-Roll Ratio for the 100 mm DMA Bearing

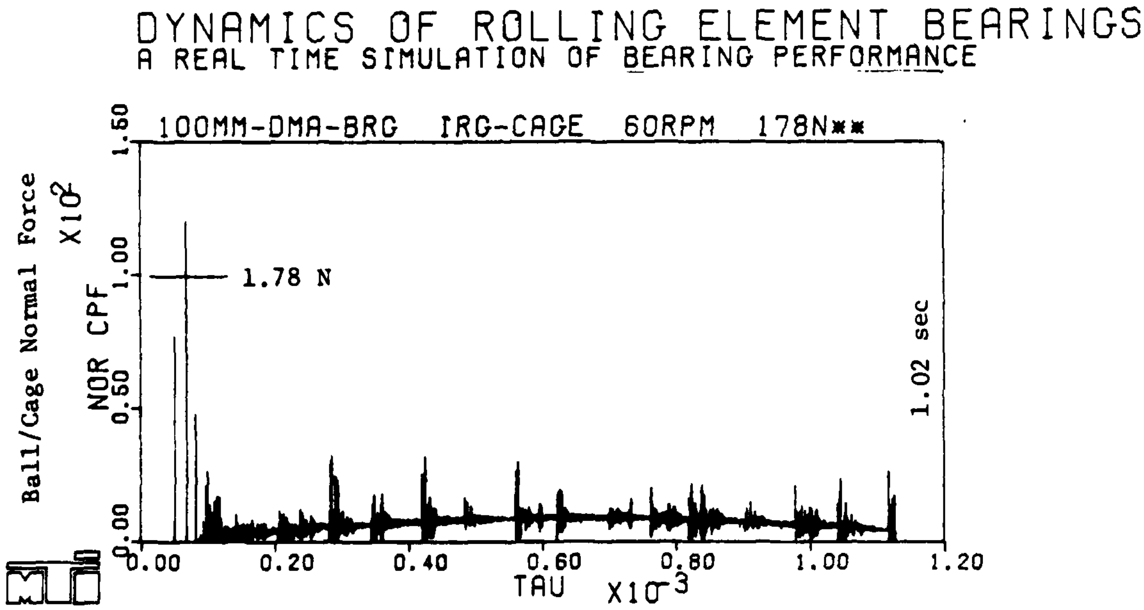

Fig. 4-11 Ball/Cage Force Variation for the 100 mm Bearing with Gravity Acting Normal to the Bearing Axis

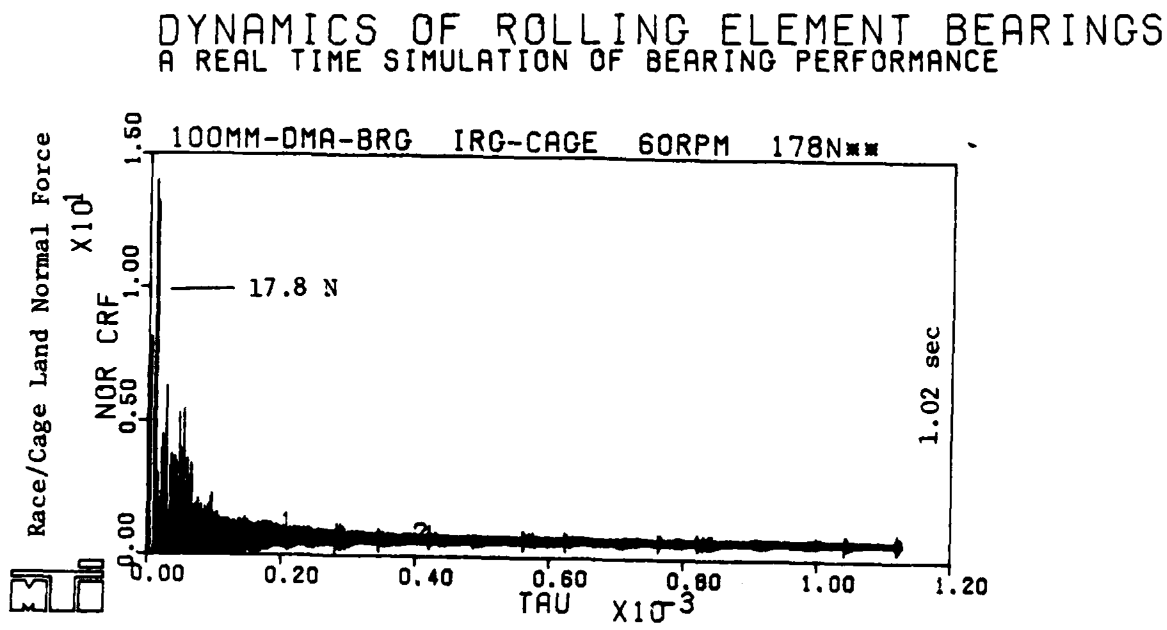

Fig. 4-12 Normal Contact Force at the Cage/Race Interface When Gravity Acts Normal to the Bearing Axis in the Case of the 100 mm DMA Bearing

from Figure 4-12, there is an appreciable difference between the contact forces at the two lands, which is due to the coning motion of the cage. This is better seen in Figure 4-13 where the cage angular accelerations are plotted. Most of this motion is clearly cyclic and it has both the high and low frequency content present in it. The high frequency corresponds to the ball/cage and cage/race elastic contacts while the low frequency is similar to what is seen earlier in Figure 4-5. Both the y and z components do show the low frequency but it is more clear in Figure 4-14, where the angular velocities are plotted. The magnitude of this low frequency is about 76.4 Hz compared to the frequency of 44.6 Hz shown earlier with the bearing more symmetrically oriented.

In spite of the complicated motion the cage does seem to be fairly stable with practically no whirl in steady-state as indicated by λ_1 in Figure 4-15. Also seen in this figure is the skid parameter λ_2, which is also fairly stable. The bearing torque variations are shown in Figure 4-16. These variations do contain both the high and low frequency cyclic variations, although they are not very clear in the figure. The ball angular velocities, as seen in Figure 4-17, are similar to those seen earlier. Also, the deviations from the race-control hypothesis are clear in terms of the spin-to-roll ratios plotted in Figure 4-18. In addition to this it may be interesting to note the ball orbital acceleration pattern shown in Figure 4-19. The noise in the ball orbital acceleration correlates well with the noise observed earlier in the skid parameter λ_2 in Figure 4-15. Again the high frequency content comes from the ball/cage elastic collisions, which are now continually varying as the ball travels around its orbit.

4.5 Performance Simulations of the 150 mm DMA Bearing

The general behavior of the 150 mm DMA bearing with ball guided cage is relatively more noisy than the 100 mm bearing. With the gravity oriented along the bearing axis the ball/cage pocket force variation is shown in Figure 4-20. Once again the mean force balances the cage weight and the ball/cage contact angle demonstrate some cyclic variations as is seen for the 100 mm bearing (Figure 4-5). The frequency of this component is now 37.4 Hz which is not very different from 44.6 Hz observed with the 100 mm bearing. The resulting cage motion is, however, substantially noisier.

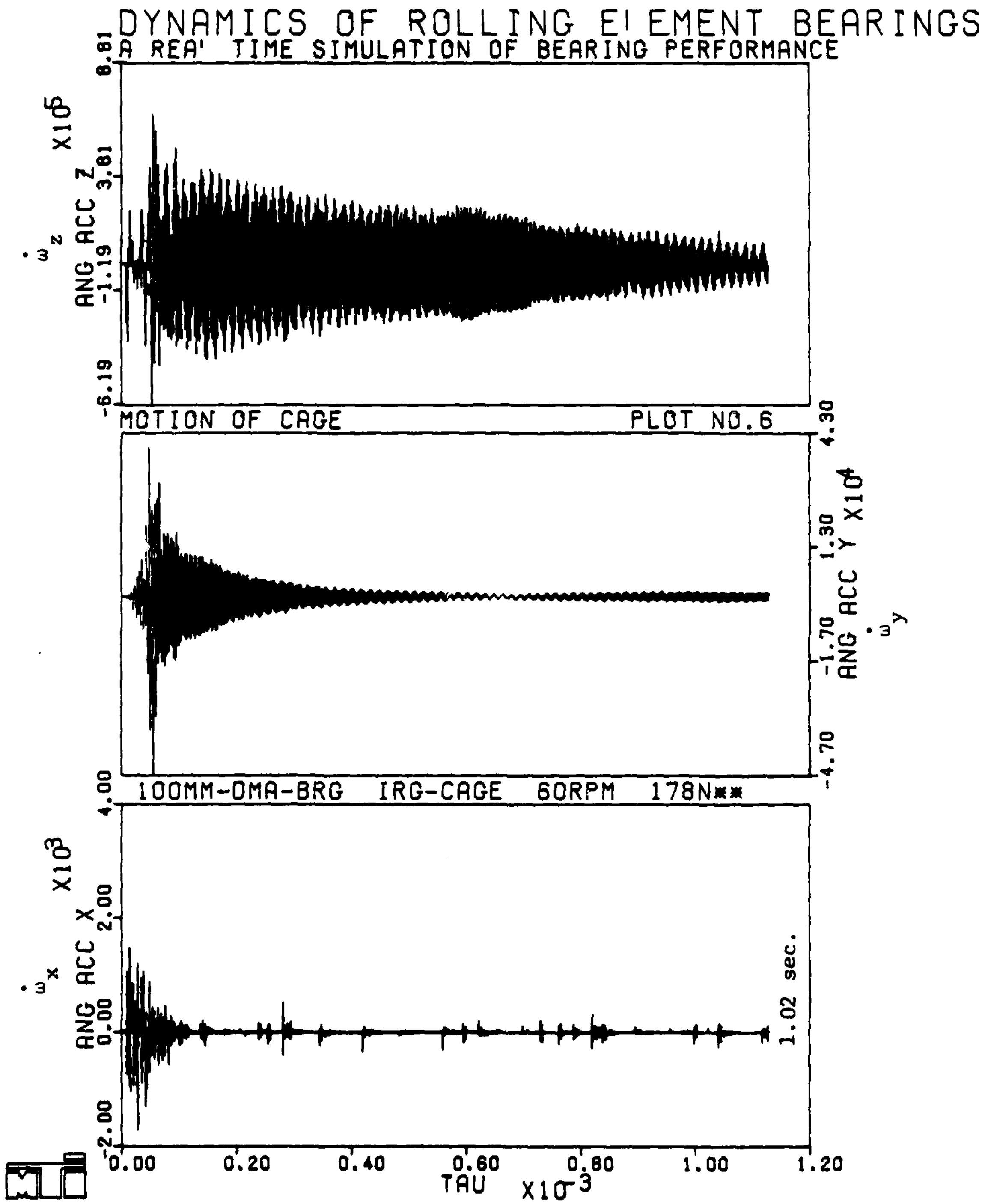

Fig. 4-13 Dimensionless Angular Accelerations of the 100 mm Bearing Cage When Gravity Acts Normal to the Bearing Axis. Scale = 1.319×10^7 RPM/S

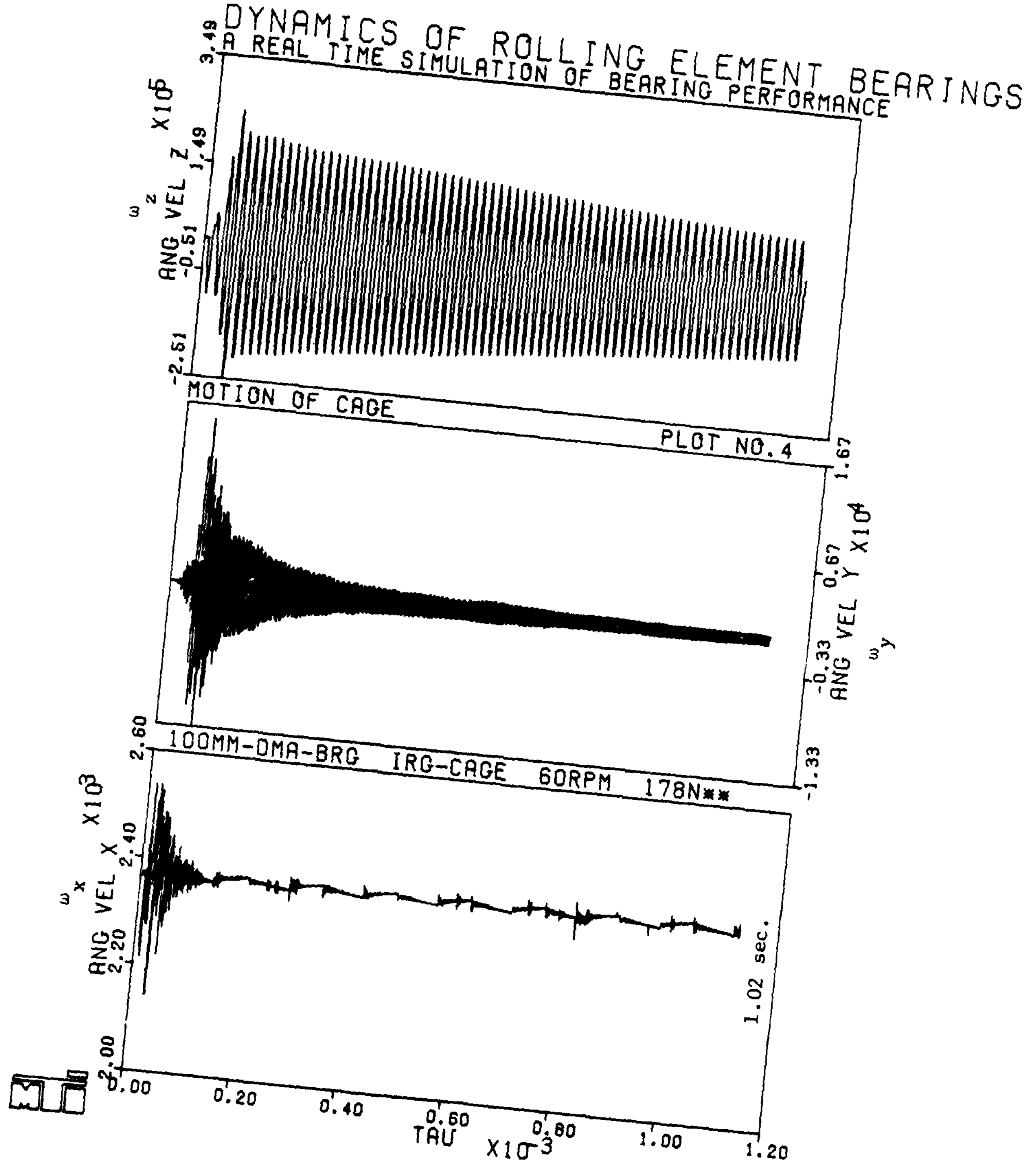

Fig. 4-14 Dimensionless Ball Angular Velocity for the 100 mm Bearing with Gravity Acting Normal to the Bearing Axis

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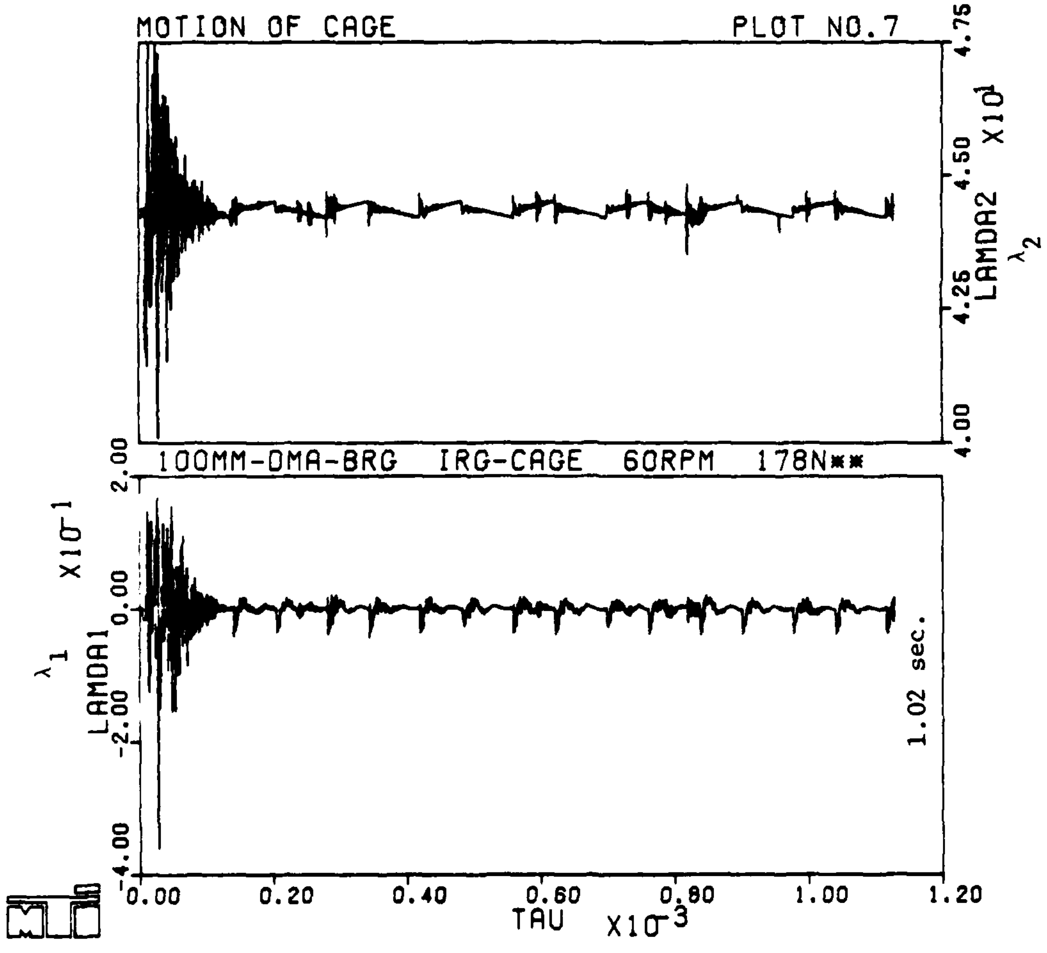

Fig. 4-15 Cage Whirl and Skid Parameters for the 100 mm Bearing with Gravity Acting Normal to Bearing Axis

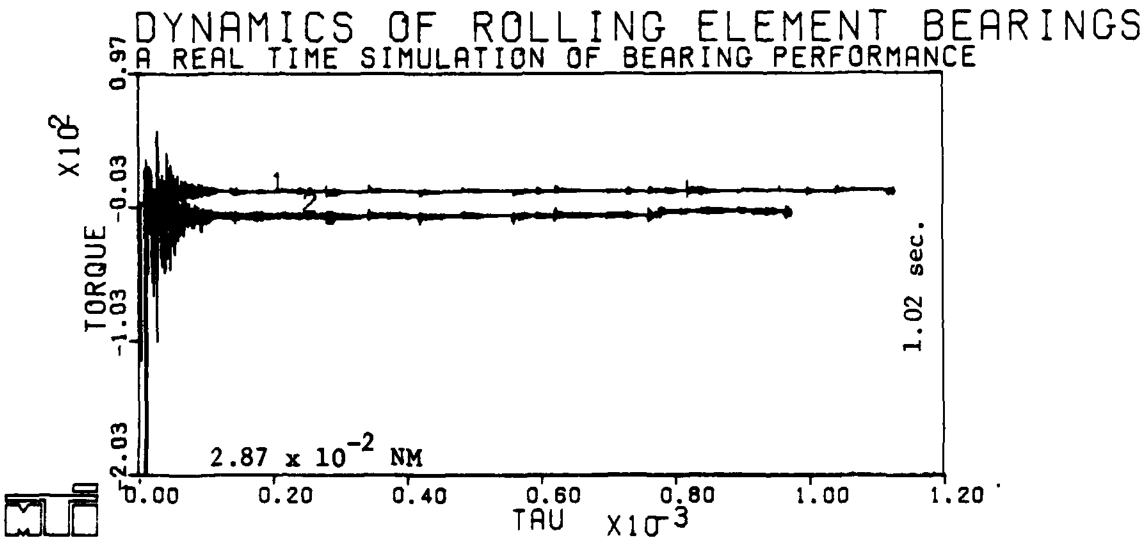

Fig. 4-16 Variation in Bearing Torque in Case of the 100 mm Bearing with Gravity Acting Normal to the Bearing Axis

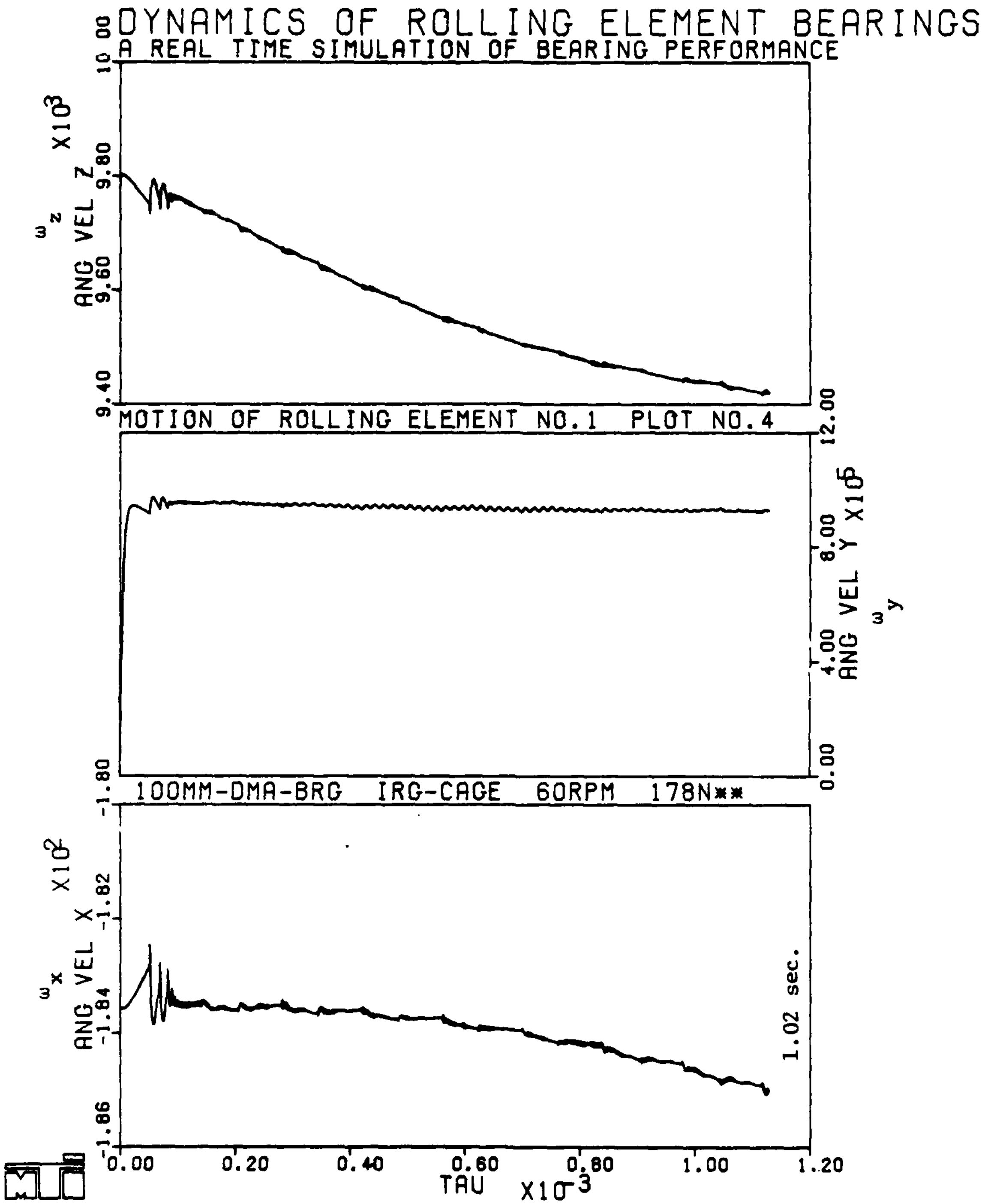

Fig. 4-17 Dimensionless Ball Angular Velocity Variation for the 100 mm DMA Bearing with Gravity Oriented Normal to the Bearing Axis

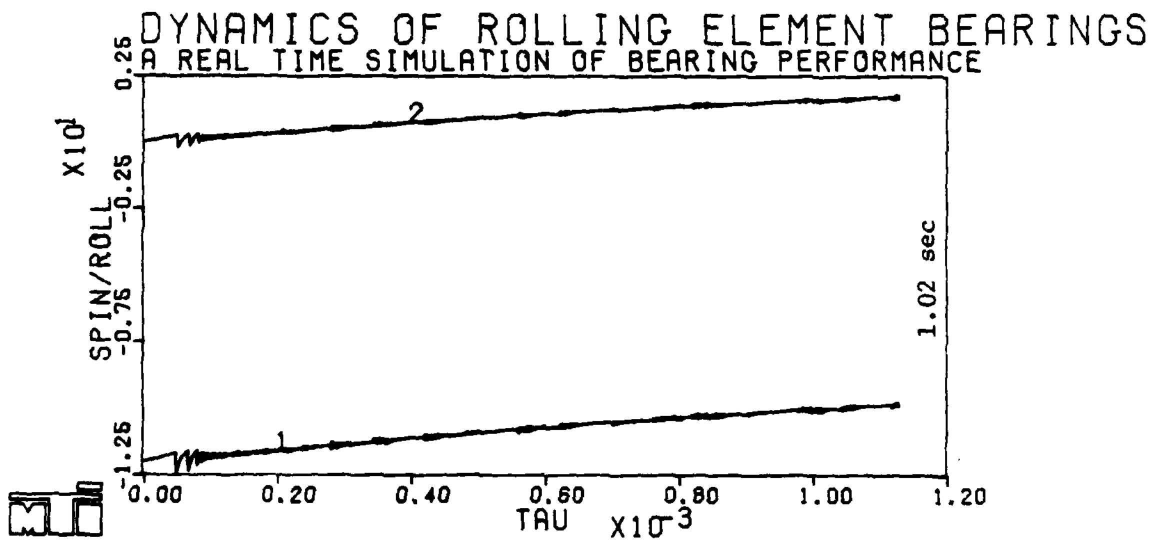

Fig. 4-18 Ball/Race Spin Variation for the 100 mm Bearing with Gravity Acting Normal to Bearing Axis

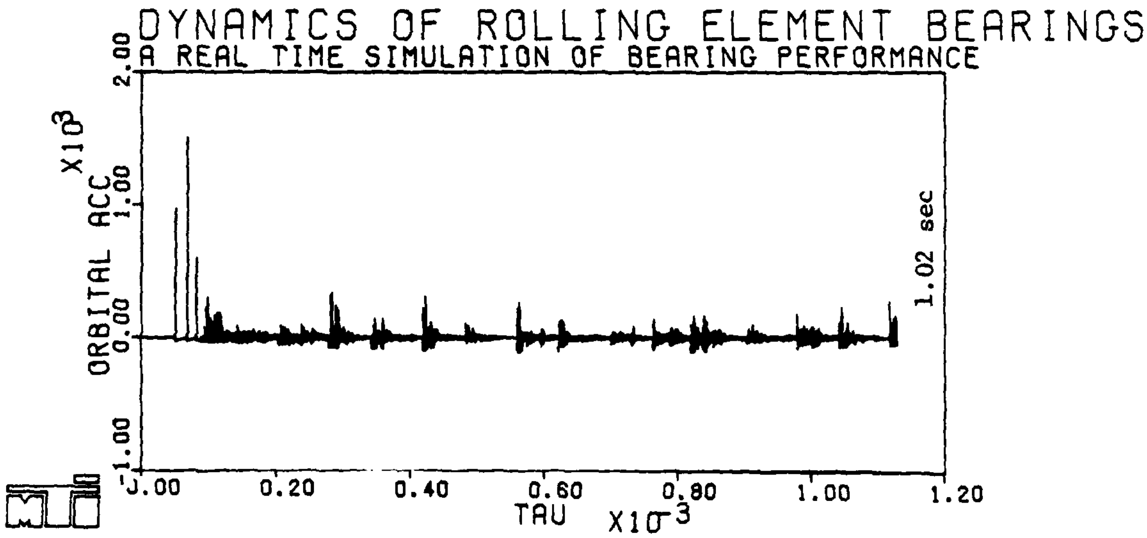

Fig. 4-19 Ball Orbital Acceleration Resulting from the Ball/Cage Collision due to Eccentric Cage in the 100 mm Bearing with Gravity Normal to Bearing Axis

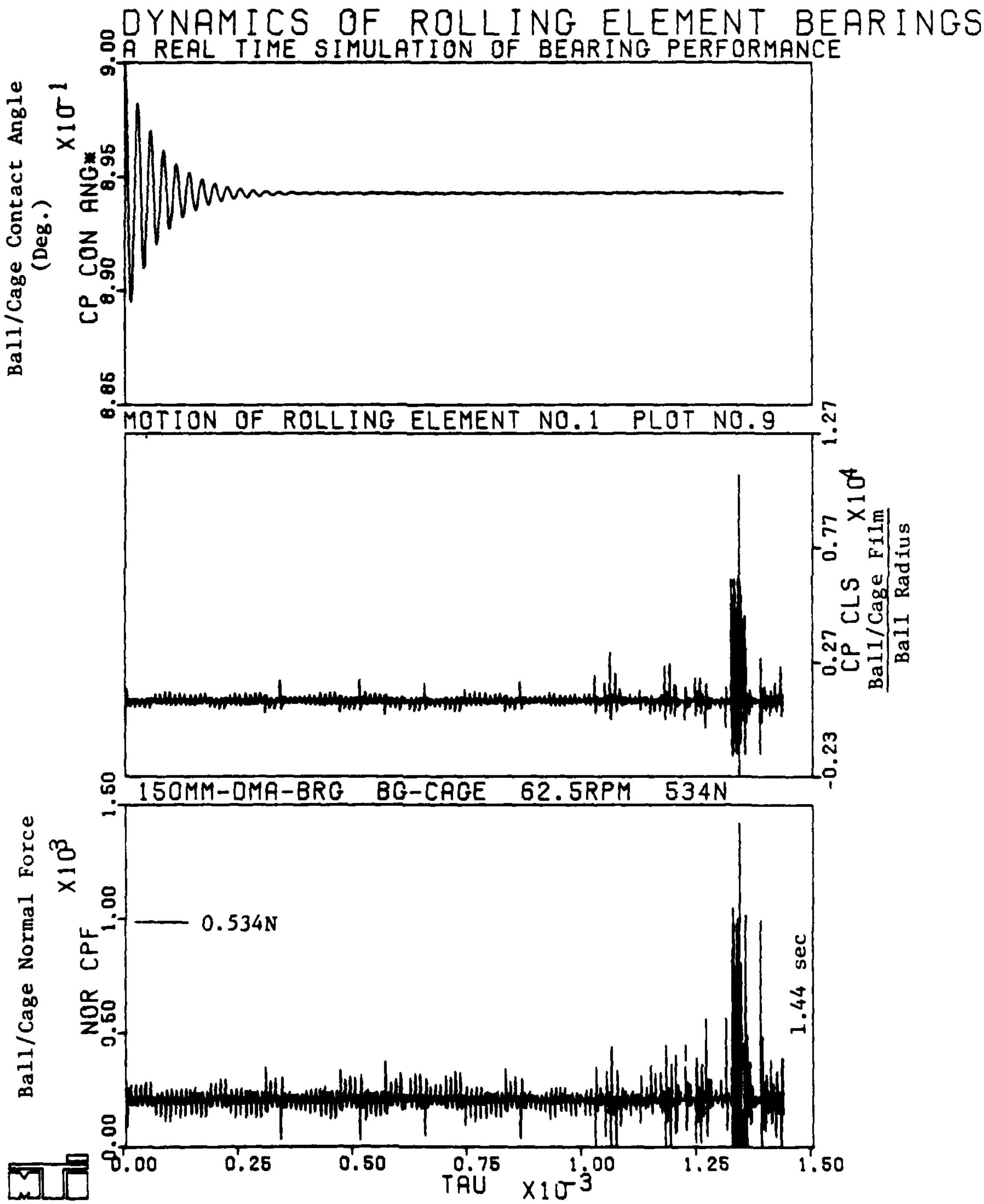

Fig. 4-20 Ball/Cage Interaction in the 150 mm Bearing With Ball Guided Cage Operating Horizontal with Gravity Acting Along the Bearing Axis

Figure 4-21 shows cage mass center accelerations. It is clear that both radial and orbital components develop beyond the threshold of machine round off errors. Although the resulting whirl motion is substantial in terms of angular velocity, the whirl radius is still small and no appreciable interaction with the pocket cone takes place. The cage whirl is indicated by λ_1 variation in Figure 4-22. It is easily seen that once the cage starts whirling the whirl velocity oscillates by large magnitudes; this may indicate some form of instability. Cage angular velocity, indicated by λ_2, is, however, fairly steady after the dampening of the initial cyclic variations. The bearing torque variations are very similar to those for the 100 mm bearing, as seen in Figure 4-23.

The general motion of the ball is almost identical to that observed in the 100 mm bearing. The ball angular velocity vector, see Figure 4-24, tends to line up with the bearing axis and the race-control hypotheses do not hold, as seen by the spin-to-roll ratios in Figure 4-25.

When the 150 mm bearing with ball-guided cage is run with gravity oriented normal to the bearing axis, significant noise in the performance simulations is produced by the repeated collisions in the cage pocket. Figure 4-26 shows typical collisions with the cylindrical part of the pockets and Figure 4-27 shows interaction at the conical guidance surface. Such repeated collisions produce considerable noise in both the ball and cage motions. The ball angular velocity variations are shown in Figure 4-28 and the orbital accelerations of the ball from the repeated impacts is shown in Figure 4-29. The angular velocity of the cage indicates substantial coning motion as seen by the y and z components in Figure 4-30. The noise generated from the ball-cage collisions is also transmitted to the cage mass center accelerations as shown in Figure 4-31. Excessive cyclic whirl of the cage is produced although no significant steady whirl is observed over the short time of this performance simulation. This is seen by the λ_1 variation in Figure 4-32. As may be expected the skid parameter, λ_2, does pick up the influence of the excessive ball/cage collisions.

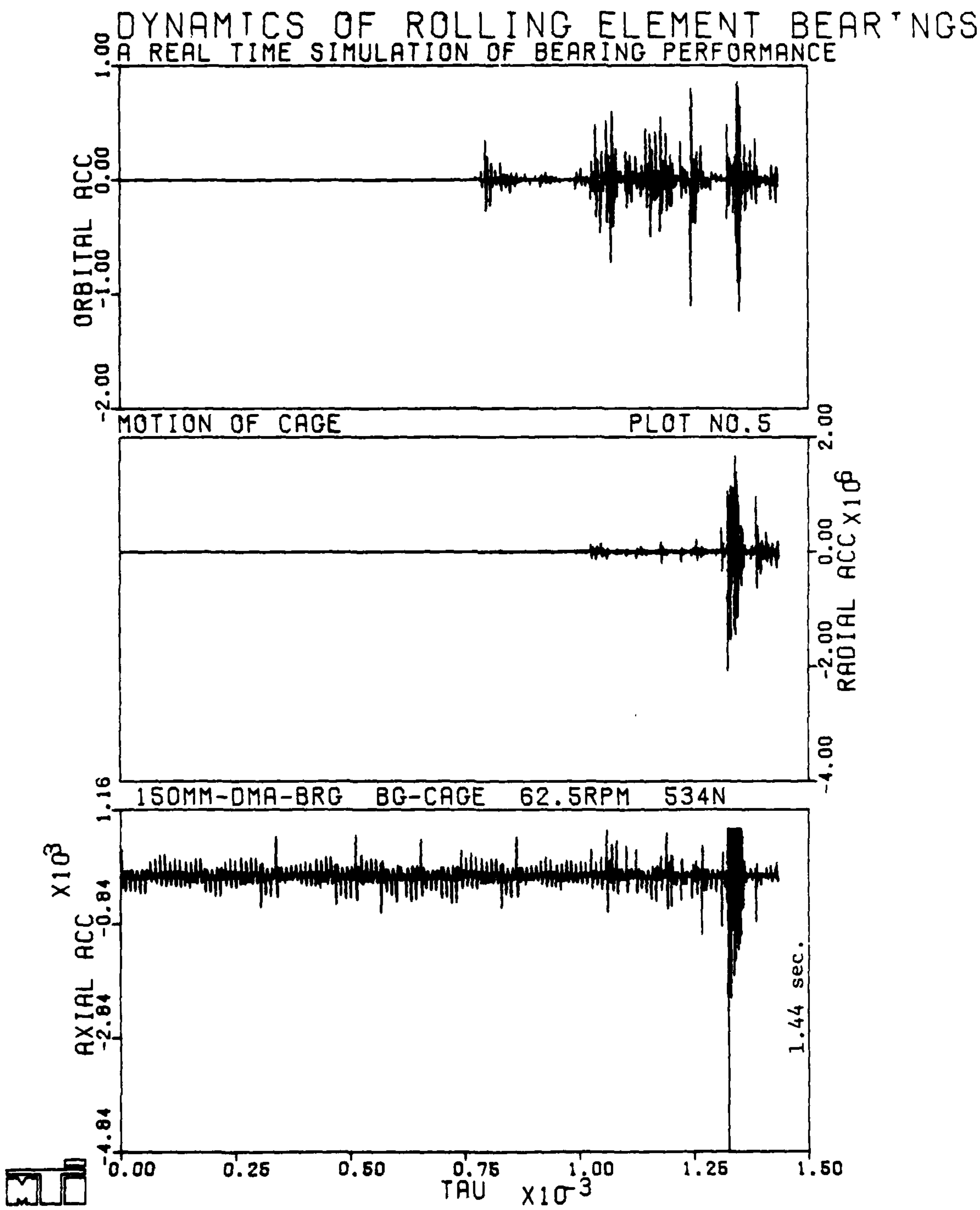

Fig. 4-21 Dimensionless Cage Mass Center Accelerations Resulting From the Noisy Ball/Cage Interaction for the 150 mm Bearing. Scale = 1.198×10^4 M/S^2, 1.03×10^7 RPM/S

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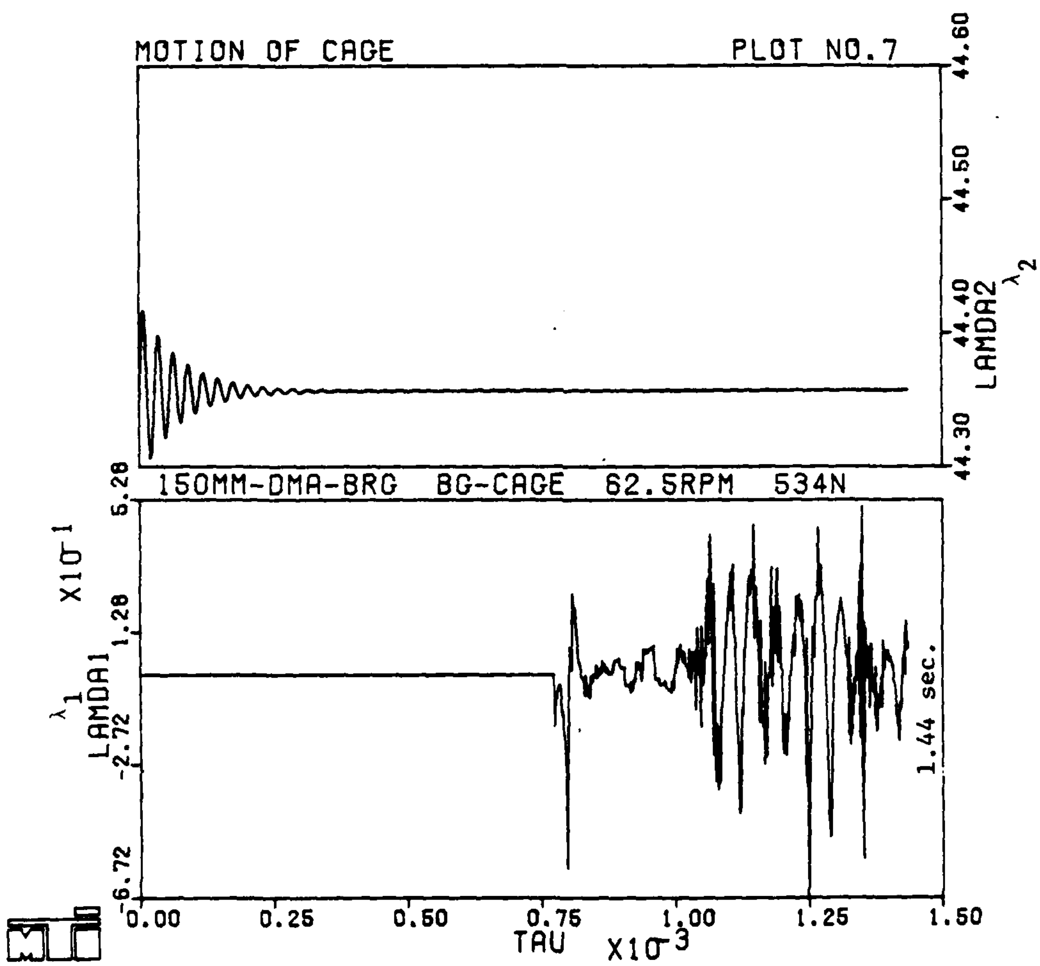

Fig. 4-22 Cage Whirl and Skid Parameter for the 150 mm DMA Bearing

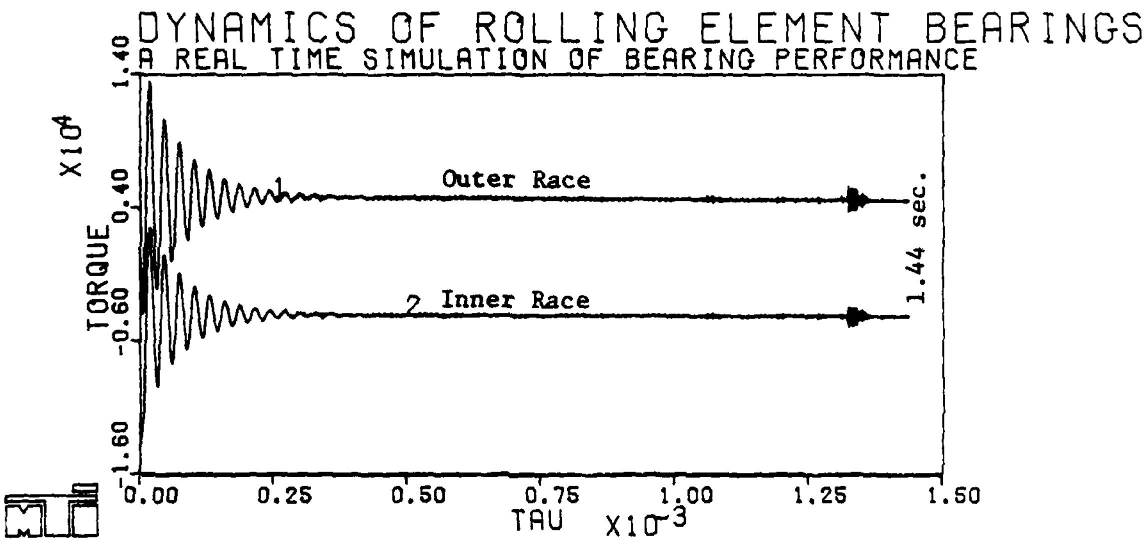

Fig. 4-23 Bearing Torque Variation for the 150 mm Bearing with Gravity Acting Along the Bearing Axis

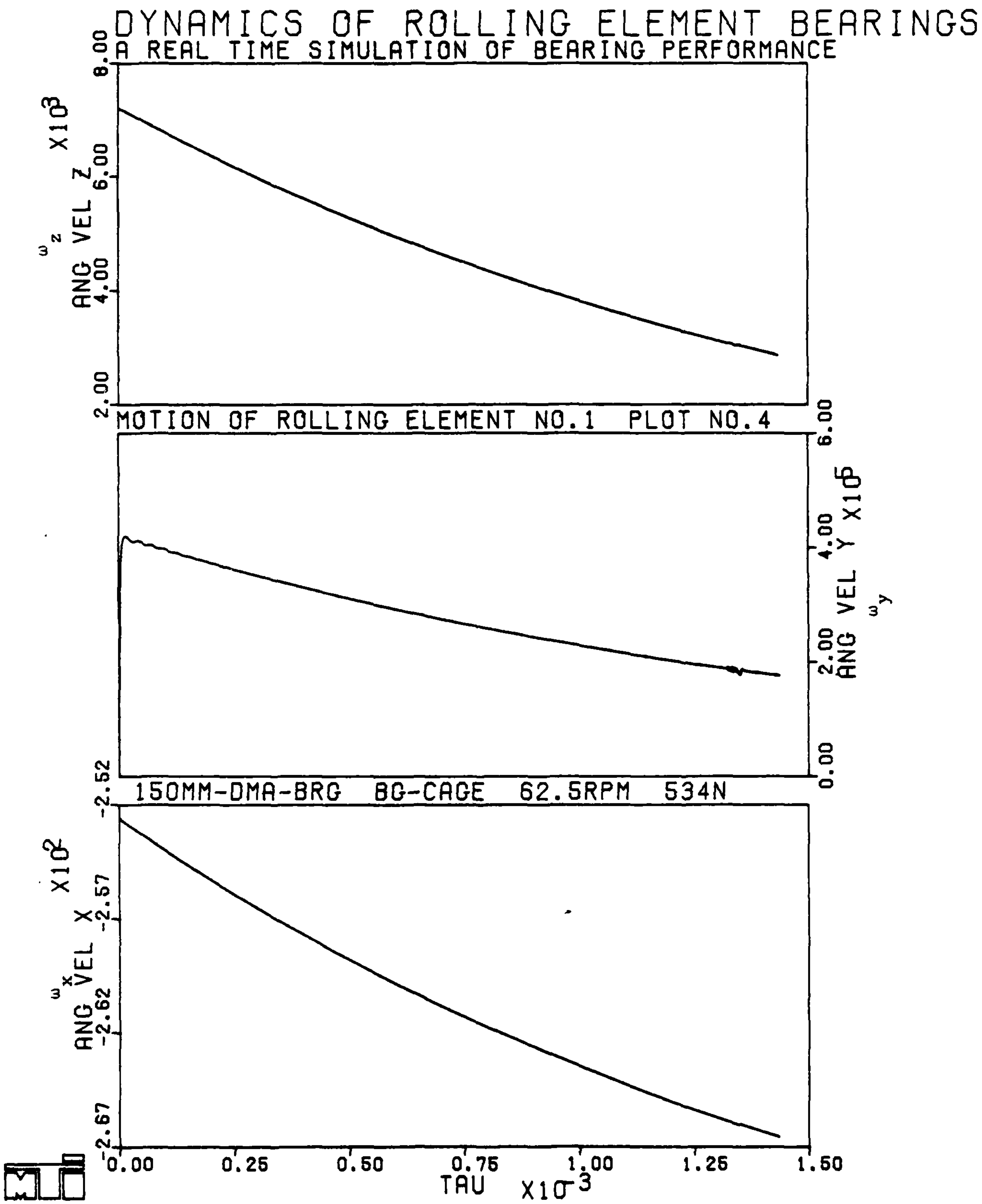

Fig. 4-24 Dimensionless Angular Velocities of the Ball in the 150 mm DMA Bearing. Scale = 9.918×10^3 RPM

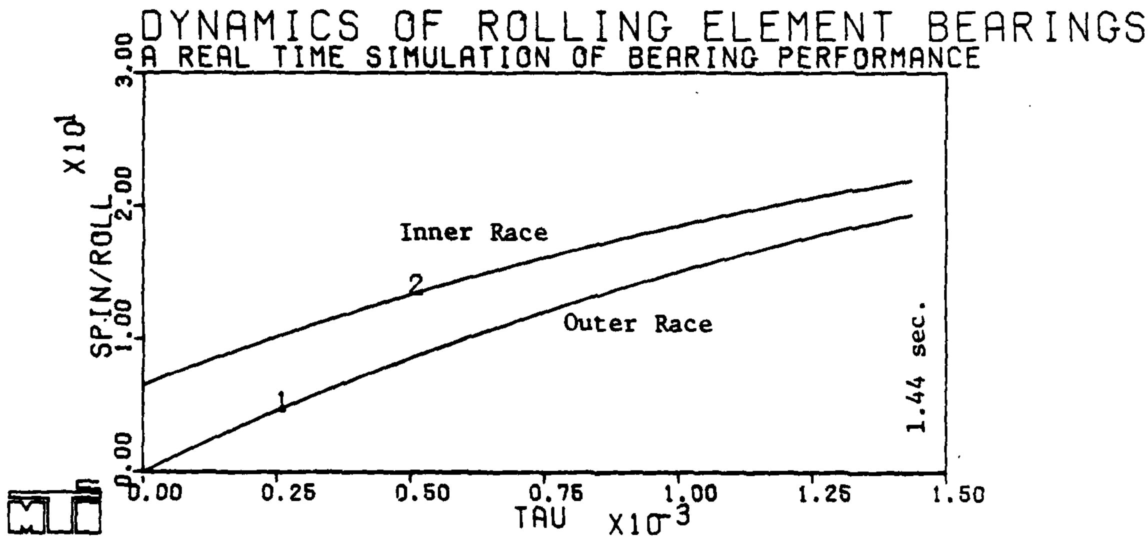

Fig. 4-25 Ball/Race Spin for the 150 mm DMA Bearing

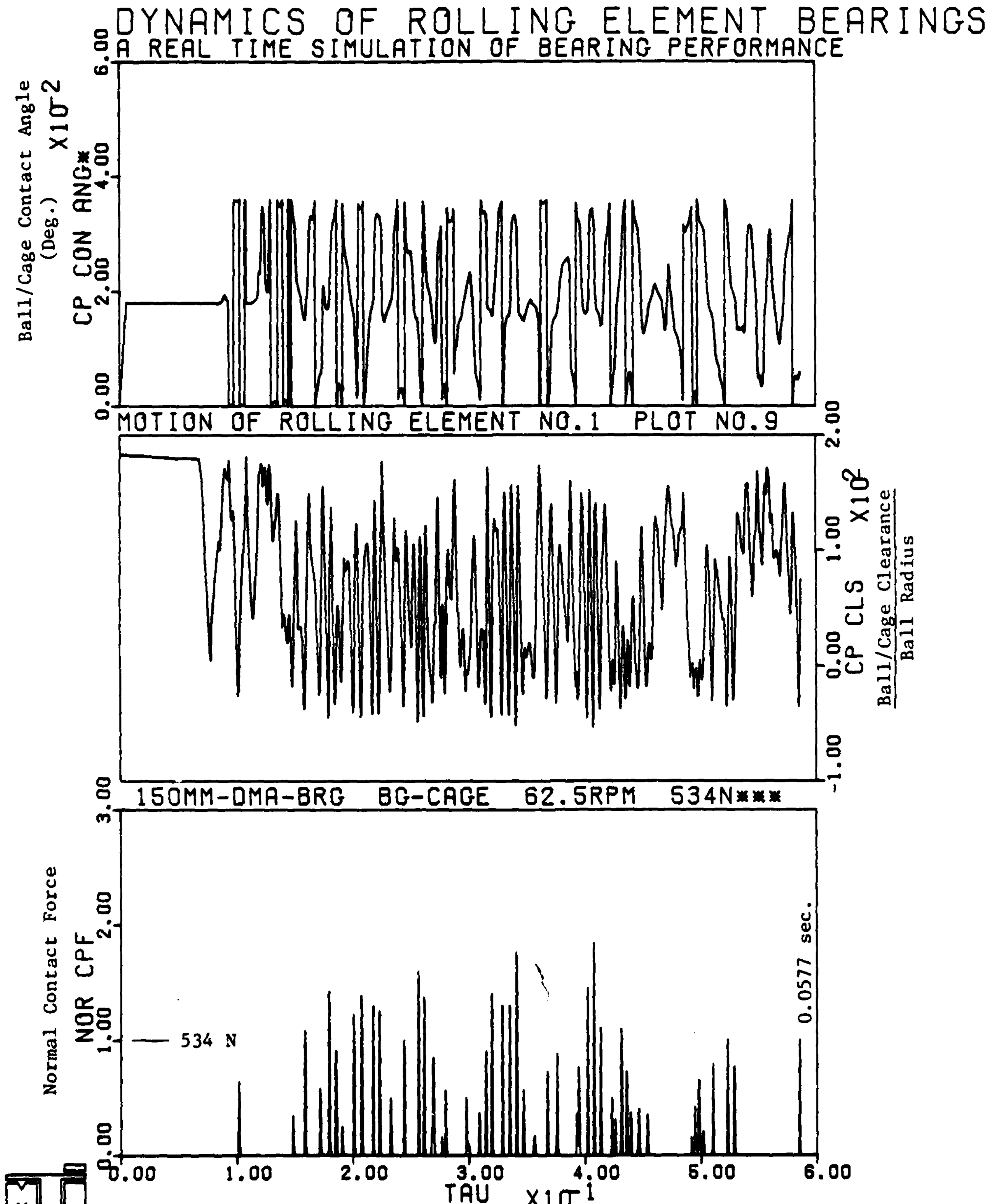

Fig. 4-26 Ball/Cage Interaction at the Cylindrical Pocket Surface for the 150 mm Bearing with Gravity Acting Normal to the Bearing Axis

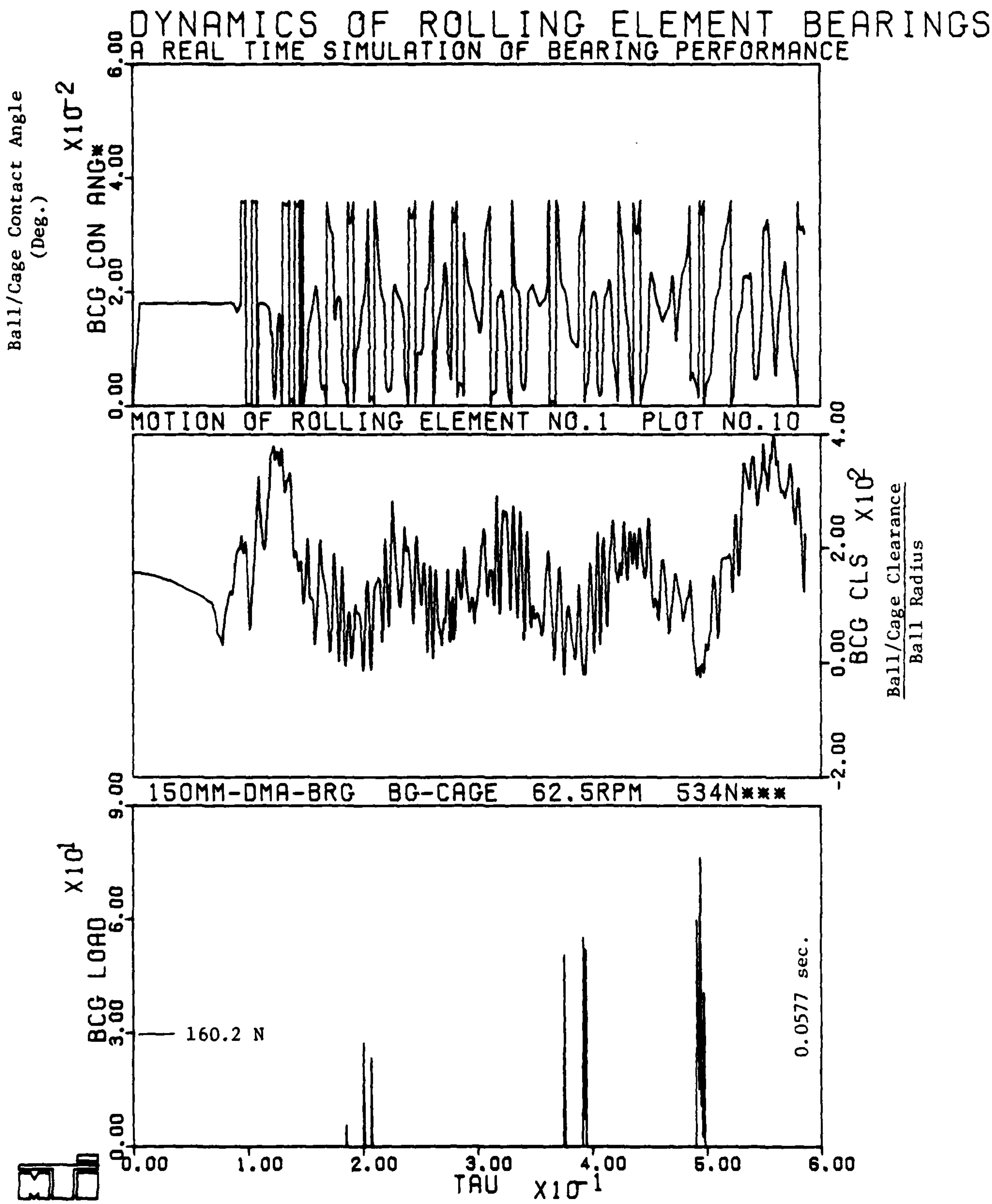

Fig. 4-27 Ball/Cage Interaction at the Conical Guidance Surface for the 150 mm Bearing with Gravity Acting Normal to Bearing Axis

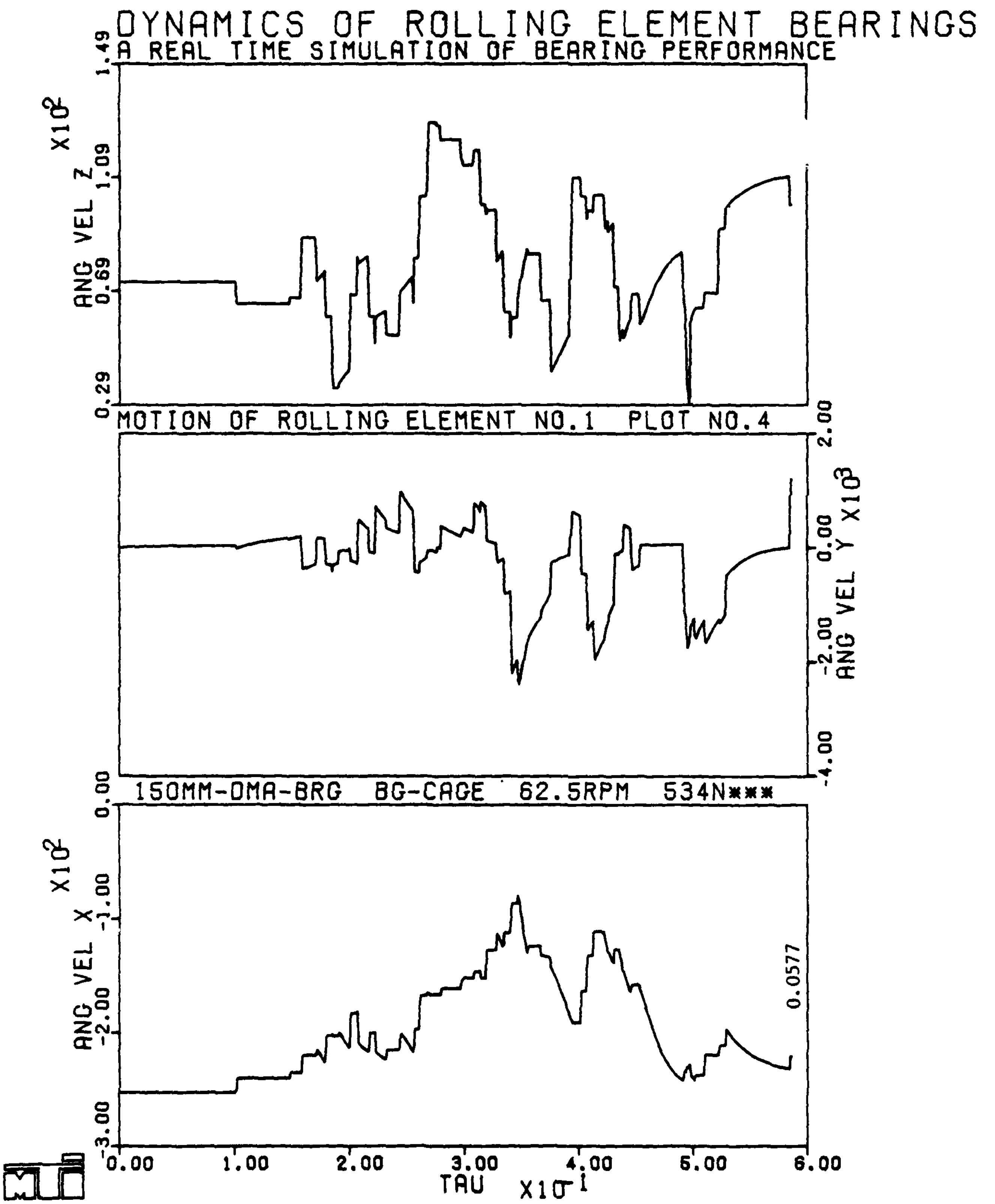

Fig. 4-28 Dimensionless Angular Velocity of the Ball for the 150 mm Bearing with Ball Guided Cage. Scale = 9.918 x 10^3 RPM

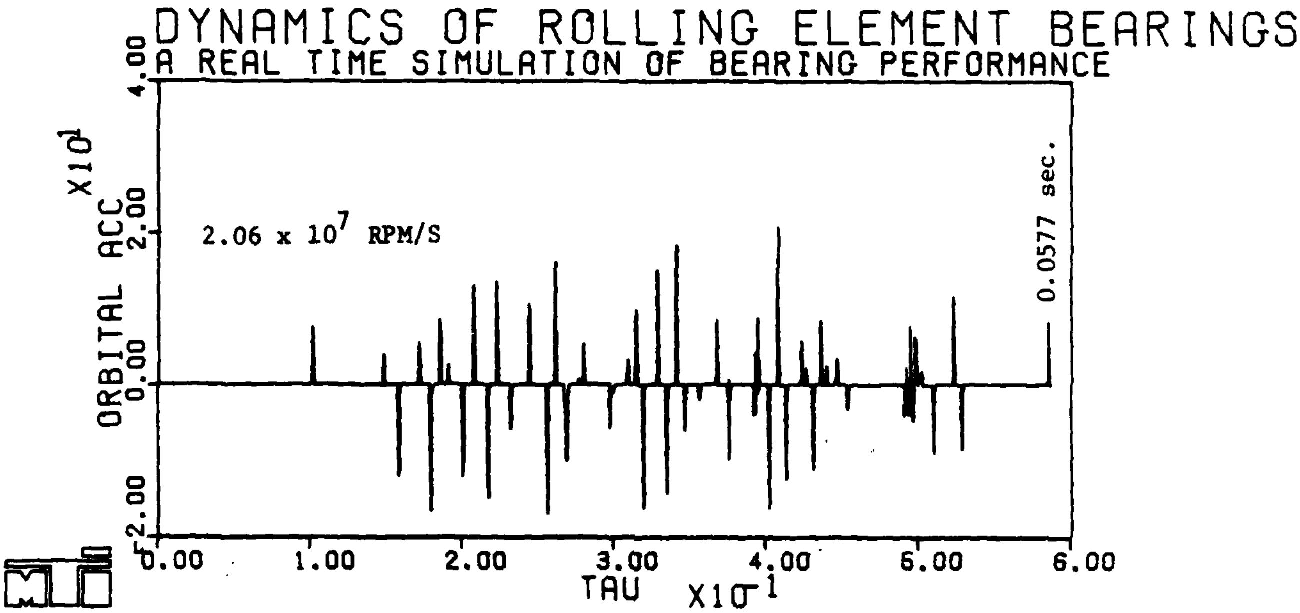

Fig. 4-29 Ball Orbital Accelerations Resulting From the Excessive Ball/Cage Interaction for the 150 mm DMA Bearing

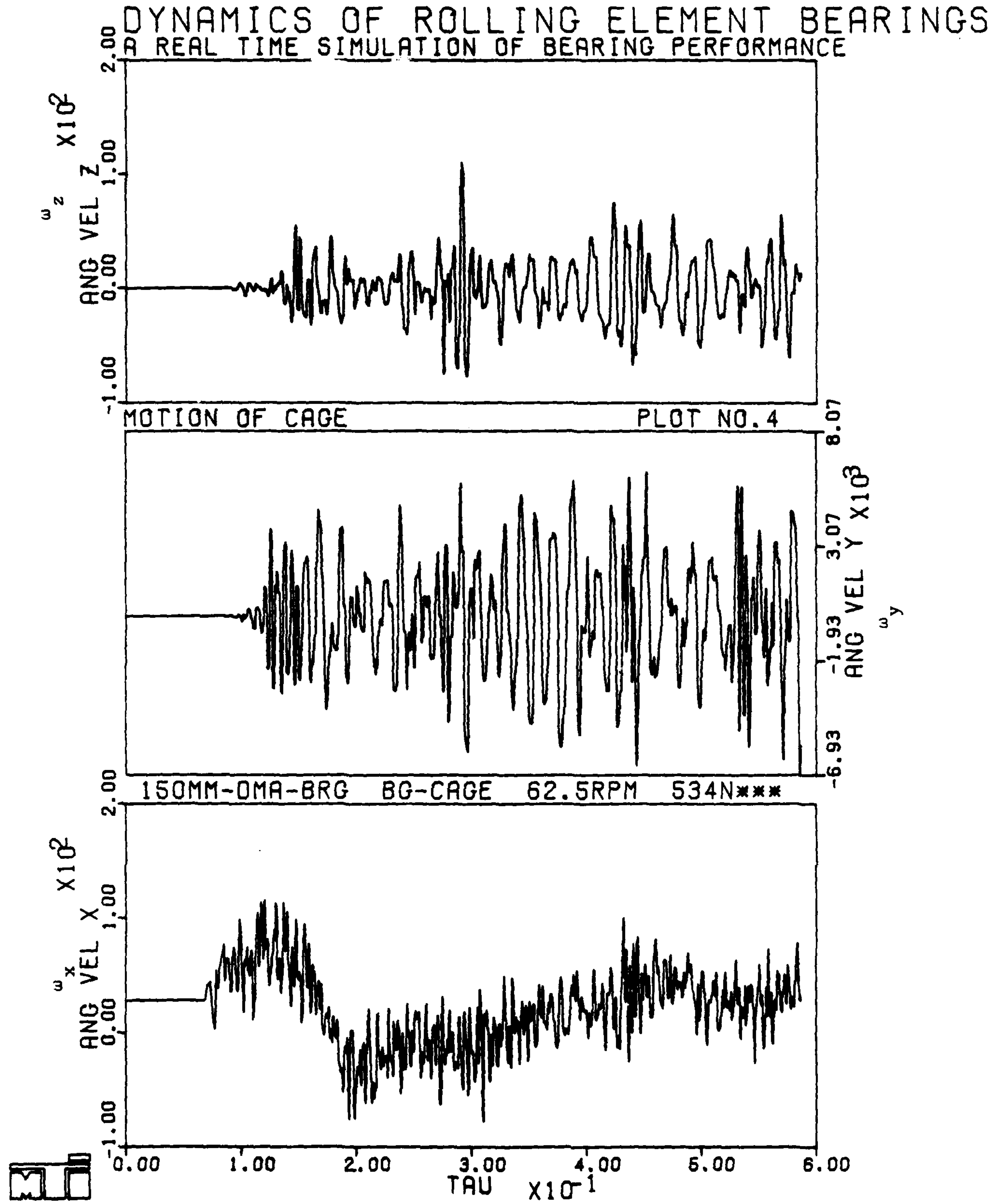

Fig. 4-30 Dimensionless Angular Velocity of the Ball Guided Cage in the 150 mm Bearing with Large Ball/Cage Interaction. Scale = 9.918×10^3 RPM

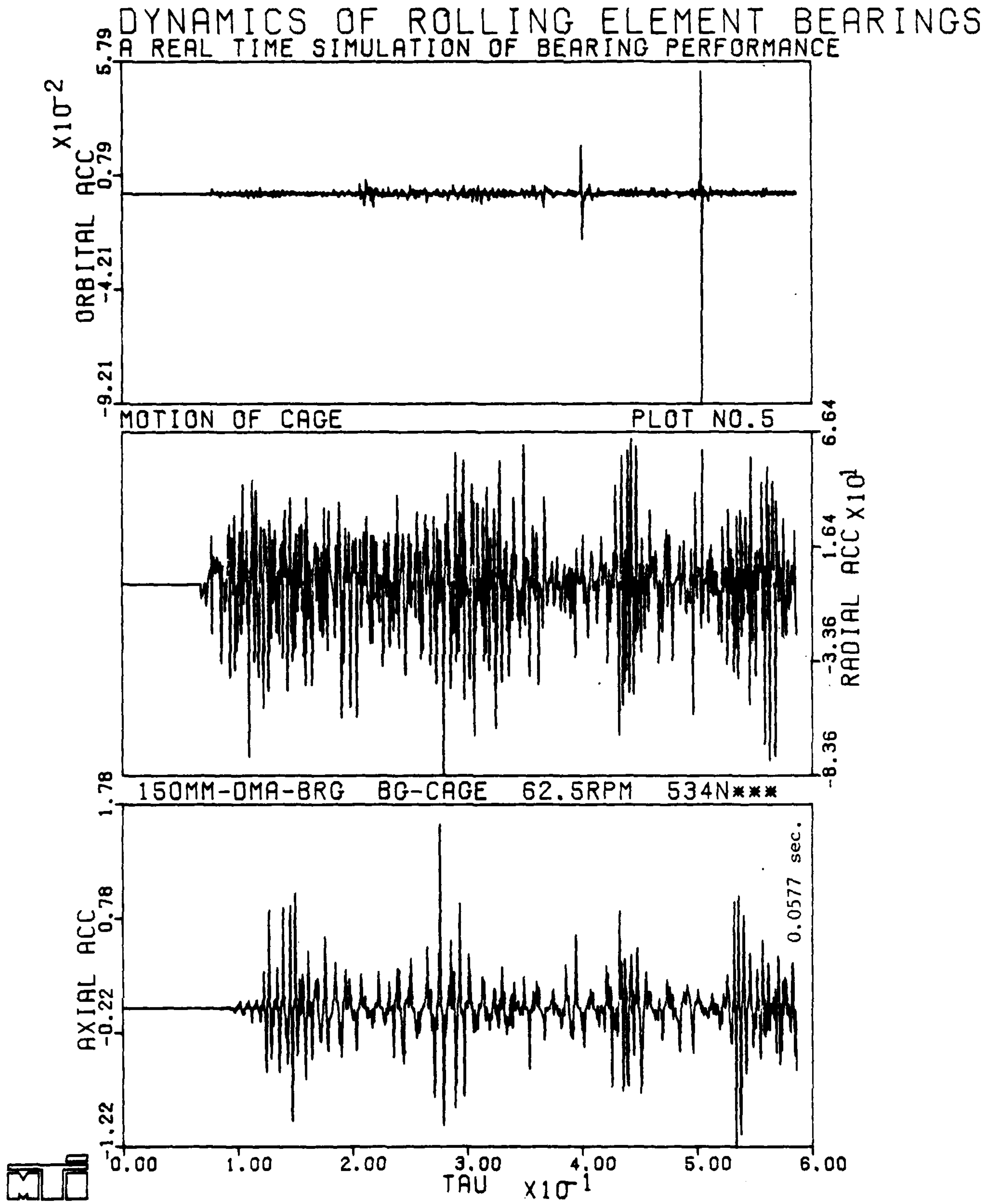

Fig. 4-31 Dimensionless Cage Mass Center Accelerations Resulting From the Excessive Ball/Cage Interaction in the 150 mm Bearing with Gravity Acting Normal to the Bearing Axis Scale = 1.199 x 10^4 M/sec^2, 1.030 x 10^7 RPM/sec

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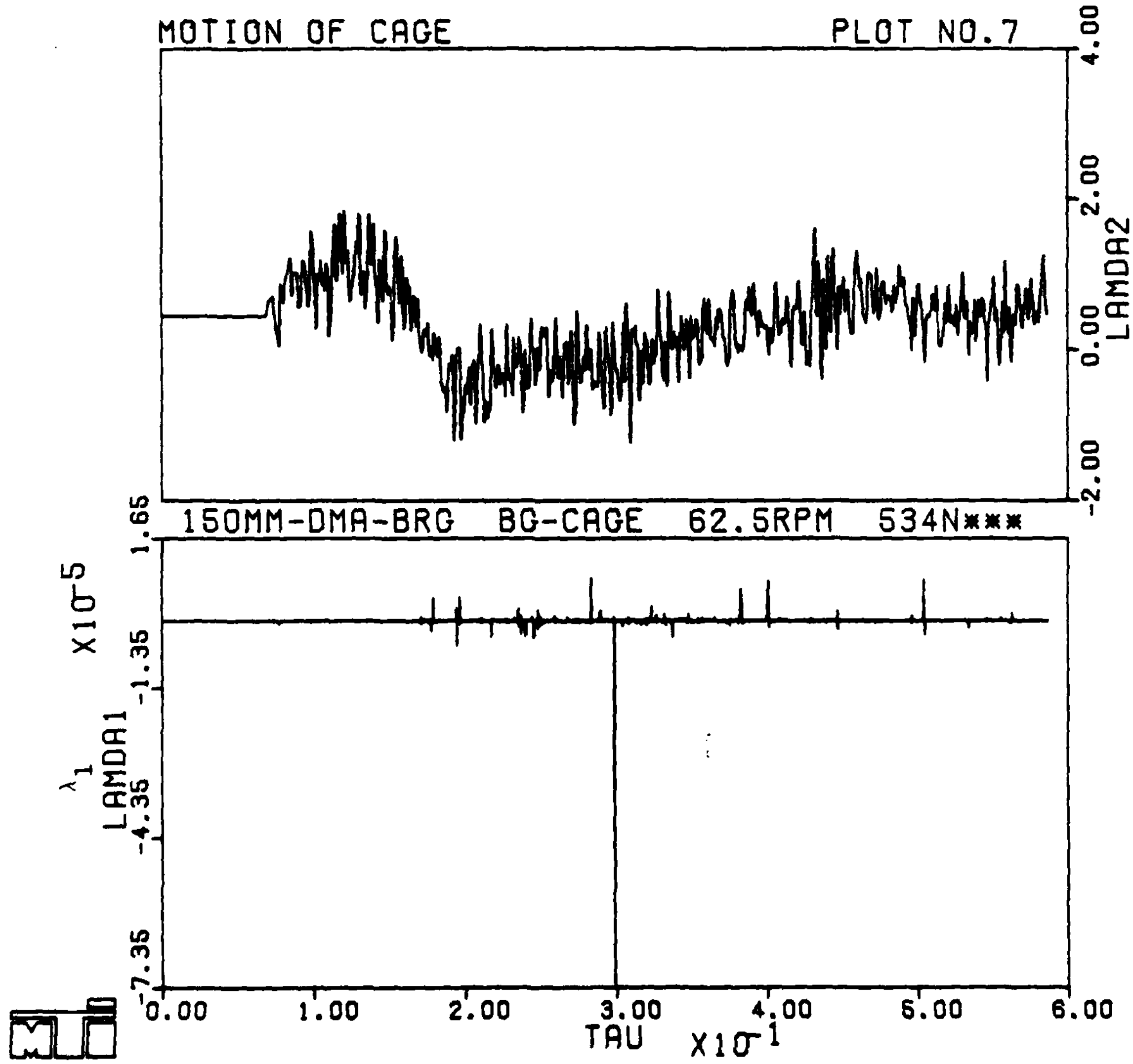

Fig. 4-32 Cage Whirl and Skid Parameters for the 150 mm Bearing with Ball Guided Cage and with Gravity Acting Normal to the Bearing Axis

5.0 ENGINE BEARING PERFORMANCE SIMULATIONS

The enhanced capabilities of RAPIDREB also apply to the high-load and high-speed engine bearings by providing bearing performance simulation over several shaft revolutions. For a given bearing and a set of operating conditions some typical performance simulations are presented in this section.

5.1 Bearing Geometry

The geometry of the bearing is shown in Figure 5-1. It is a 100 mm bore ball bearing with a steel cage guided on the inner race. The design corresponds to an actual test bearing at the Air Force Propulsion Laboratories.

5.2 Lubricant Traction Models

Conventional lubricant with the MIL-L-7808 specification is assumed. The traction behavior of this lubricant has been studied fairly extensively and the model in the original DREB program has been demonstrated to show a reasonable fit with the experimental data. The model is also built into RAPIDREB and appropriate input option is exercised to select the MIL-L-7808 model.

For the ball/cage and cage/race interfaces a hypothetical traction model (see Appendix C) is used to compute the friction forces when a metal contact takes place.

5.3 Operating Conditions

To simulate typical test conditions, the following operating conditions are assumed:

Axial load =	18,000 N
Radial load =	4,500 N
Inner race speed =	20,000 RPM
Outer race speed =	0
Operating temperature =	330°K

Gravity acts normal to the bearing axis, but in this case it really plays no appreciable role because the applied forces are quite high. Radial and axial equilibrium contraints on ball motion are assumed to eliminate the high frequency ball/race vibrations. Also the quasi-static solutions are used to determine the initial conditions.

```
RRRRRRRRRRRRRRR    DDDDDDDDDDDDDDD    RRRRRRRRRRRRRRR    EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
RRRRRRRRRRRRRRRR   DDDDDDDDDDDDDDDD   RRRRRRRRRRRRRRRR   EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBBB
  RR         RR      DD         DD      RR         RR    EE                    BB          BB
  RR         RR      DD         DD      RR         RR    EE                    BB          BB
  RRRRRRRRRRRRR      DD         DD      RRRRRRRRRRRRR    EEEEEEEEEE            BBBBBBBBBBBB
  RRRRRRRRRRRR       DD         DD      RRRRRRRRRRRR     EEEEEEEEEE            BBBBBBBBBBBB
  RR       RR        DD         DD      RR       RR      EE                    BB          BB
  RR        RR       DD         DD      RR        RR     EE                    BB          BB
  RR         RR    DDDDDDDDDDDDDDDD     RR         RR    EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBBB
  RR          RR   DDDDDDDDDDDDDDD      RR          RR   EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
```

R A P I D   S I M U L A T I O N   O F   T H E

D Y N A M I C S   O F   R O L L I N G   E L E M E N T   B E A R I N G S

-A REAL TIME PERFORMANCE SIMULATION-
(VERSION RAPIDREB.0)

PRADEEP K. GUPTA
MECHANICAL TECHNOLOGY INCORPORATED
968 ALBANY-SHAKER ROAD
LATHAM, NEW YORK
U. S. A.

BEARING TYPE -- BALL   SPEC CODE -- 100MM ENGINE BRG MOD CAGE 18/4.5KN 20KRPM

BEARING GEOMETRY

BORE	(M) =	1.00000E-01	OUTER RACE SHRINK FIT	(M)	=	1.00000E-05
OUTSIDE DIAMETER	(M) =	1.80000E-01	INNER RACE SHRINK FIT	(M)	=	5.00000E-05
SHAFT INNER DIA	(M) =	2.00000E-02	HOUSING OUTER DIA	(M)	=	2.05000E-02
BALL DIAMETER	(M) =	1.90500E-02	NUMBER OF BALLS		=	18
PITCH DIAMETER	(M) =	1.40000E-01	OUTER RACE CUR FACTOR		=	5.20000E-01
CONTACT ANGLE	(DEG) =	2.50000E+01	INNER RACE CUR FACTOR		=	5.40000E-01
END PLAY	(M) =	9.66105E-04	DIAMETRAL PLAY	(M)	=	2.14180E-04
CAGE OUTER DIA	(M) =	1.48800E-01	CAGE OUTER DIA CLS	(M)	=	7.00000E-03
CAGE INNER DIA	(M) =	1.29700E-01	CAGE INNER DIA CLS	(M)	=	3.00000E-03
EFF CAGE WIDTH	(M) =	2.73000E-02	DIA RE/CAGE CLEARANCE	(M)	=	8.26000E-04

	GUIDANCE TYPE	GUIDING RACE RADIUS (M)	GUIDING CAGE RADIUS (M)	EFF LAND WIDTH (M)	CAGE HALF WIDTH (M)
GUIDING LAND I	2	6.42350E-02	6.48500E-02	4.00000E-03	1.36500E-02
GUIDING LAND II	2	6.42350E-02	6.48500E-02	4.00000E-03	1.36500E-02

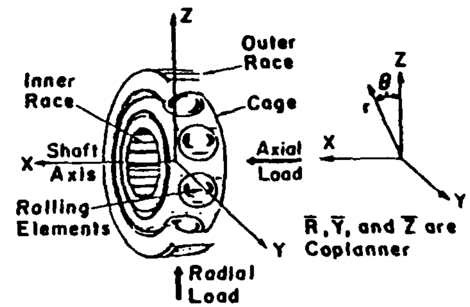

Fig. 5-1 Geometrical Description of the 100 mm Engine Bearing with Inner Race Guided Cage

5.4 Performance Simulations

Performance simulations for the engine bearing at the above conditions are obtained over seven shaft revolutions and hence definite steady state behavior can be easily understood. Typical initial parameters are illustrated in the computer output presented in Appendix C.

As might be expected in case of combined axial and radial load, the ball load and contact angles go through a cyclic variation with a frequency corresponding to the ball orbital velocity. This is seen in Figure 5-2, where over three wavelengths of variations corresponding to over seven shaft revolutions are shown. It is also seen in this figure that the race-control hypotheses do not hold here also and definite spin velocities on both races develop in steady-state. Ball angular velocities demonstrate an interesting effect, see Figure 5-3. Since the initial quasi-static solutions do not allow gyroscopic slip and exact equilibrium of moments is not satisfied, the ball immediately tends to slip about the transverse y axis due to the gyroscopic moments. This alters the x and z components also and it takes approximately two shaft revolutions for the angular velocities to develop some steady state pattern and satisfy the gyroscopic slip as permitted by the lubricant characteristics. The ball cage interactions show that about one collision in each pock takes place per revolution of the cage and the ball drives the cage (ball/cage contact angle of 180°) for part of the revolution and gets driven by the cage (contact angle of zero) for the remaining part. This is seen in Figure 5-4. The ball/cage forces seen in this figure are really the collision forces and the hydrodynamic forces, being very small compared to the collision forces, are not seen in the figure. However, the ball/cage approach relative to the radial pocket clearance clearly demonstrates the transition from hydrodynamic to metal contact.

The race/cage interaction is very dynamic and interesting to note. Figure 5-5 shows the race/cage force variations at the two lands (labelled 1 and 2) on either side of the balls. Intially both lands show identical collision forces but when appreciable coning motion of the cage develops, the forces on the two lands begin to differ; this is more clear from the traction

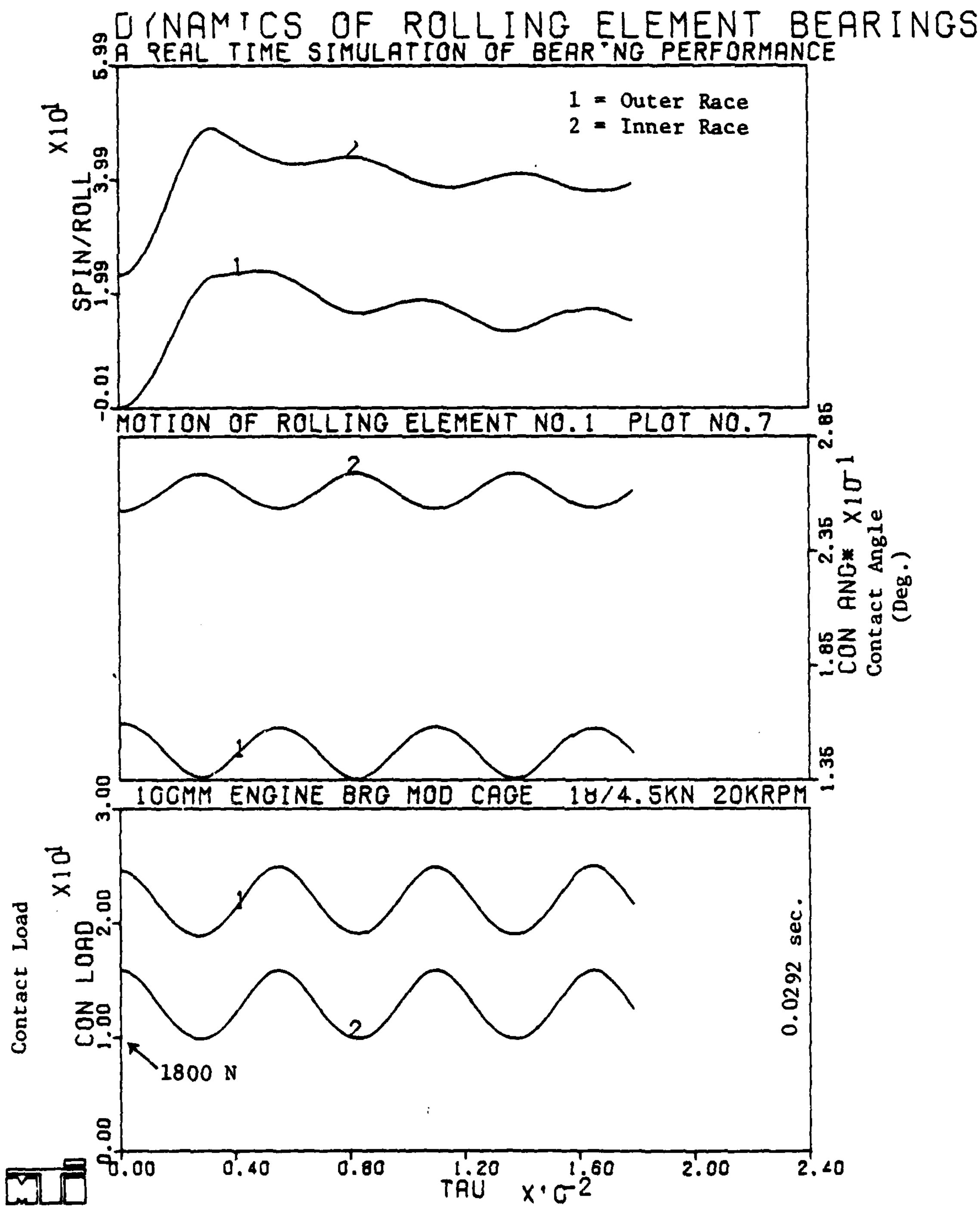

Fig. 5-2 Variations in Ball/Race Load, Contact Angle and Spin-to-Roll for the 100 mm Engine Bearing

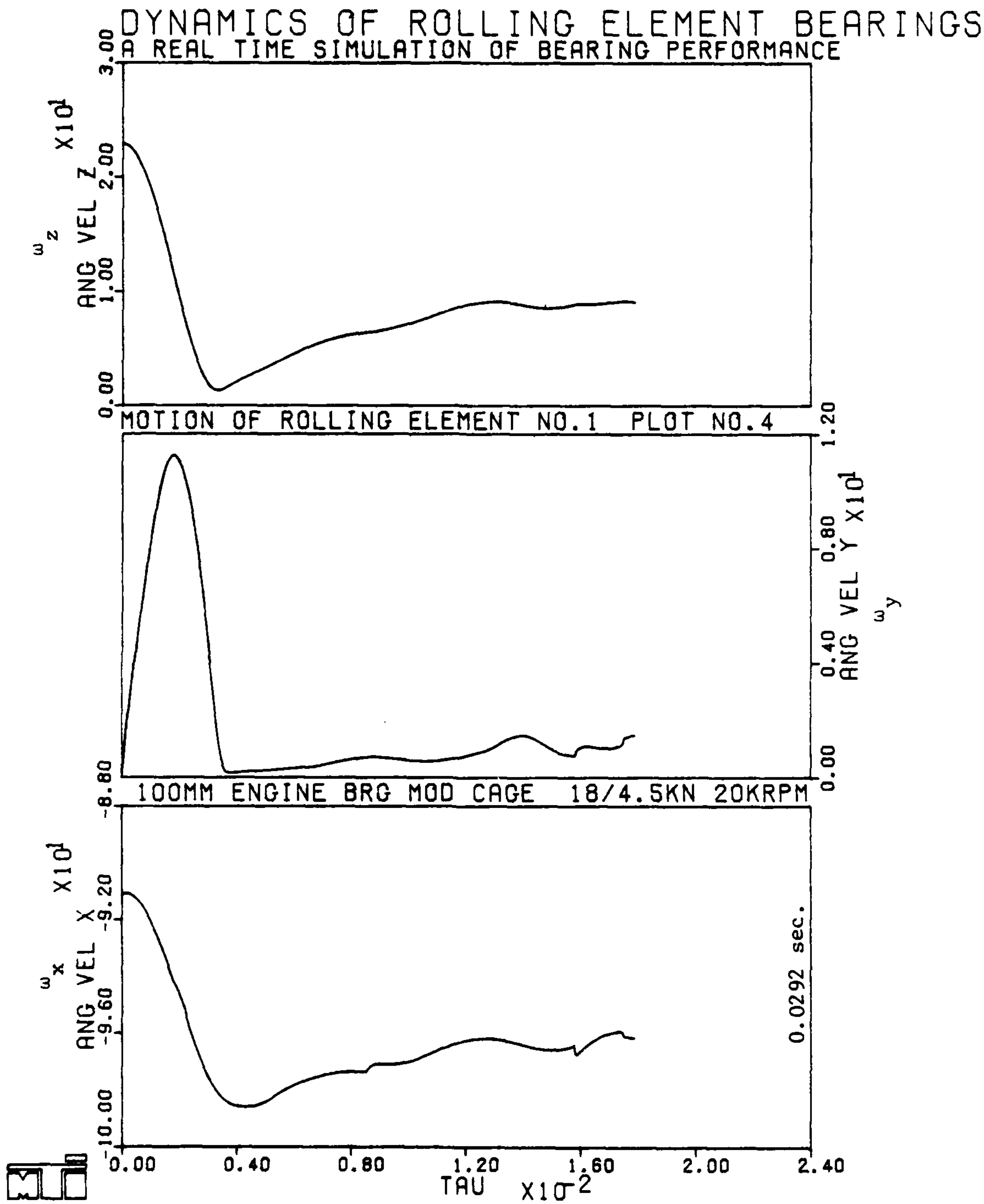

Fig. 5-3 Dimensionless Ball Angular Velocity Variation for the 100 mm Engine Bearing. Scale = 7.84×10^4 RPM

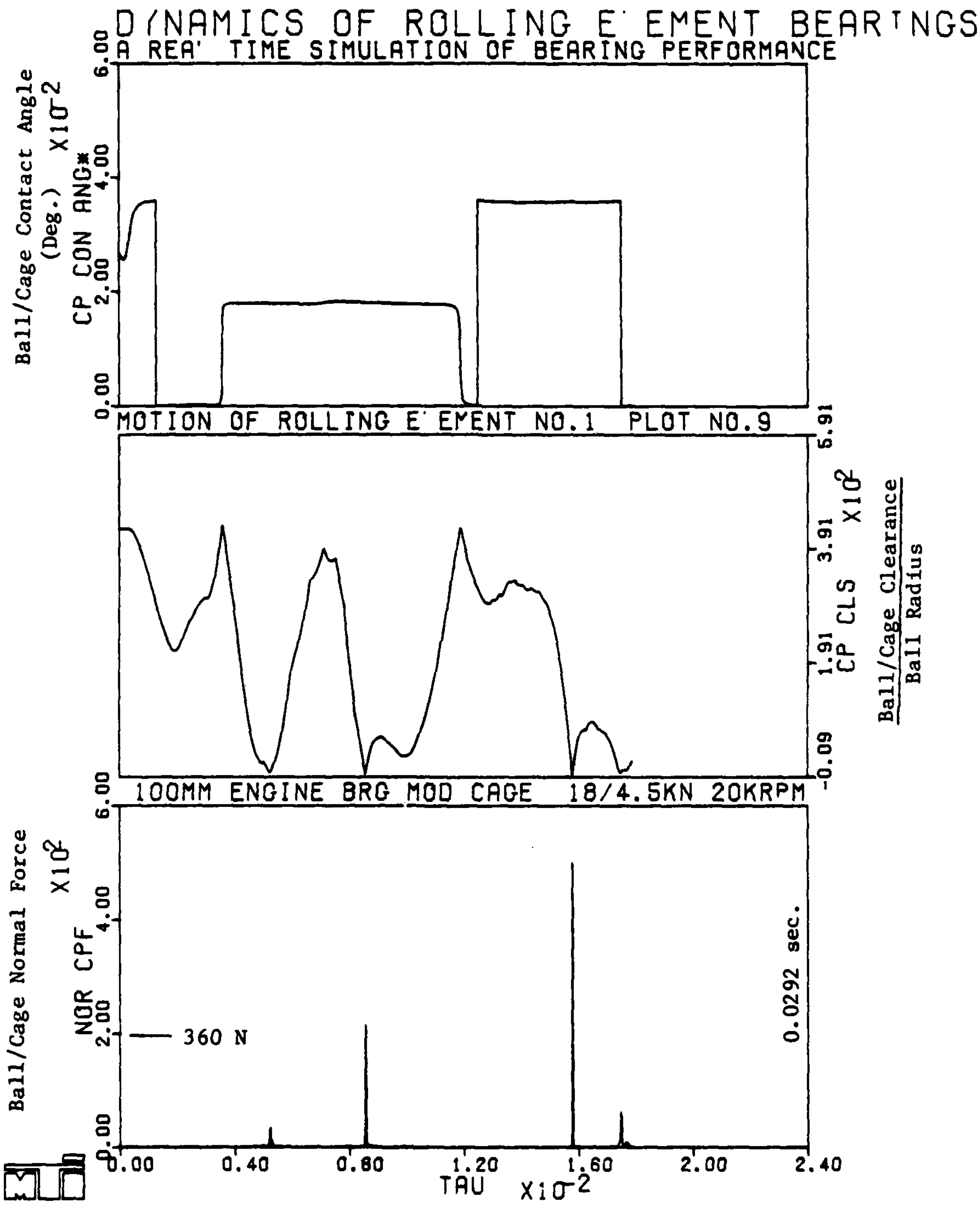

Fig. 5-4 Ball/Cage Interaction for the 100 mm Engine Bearing

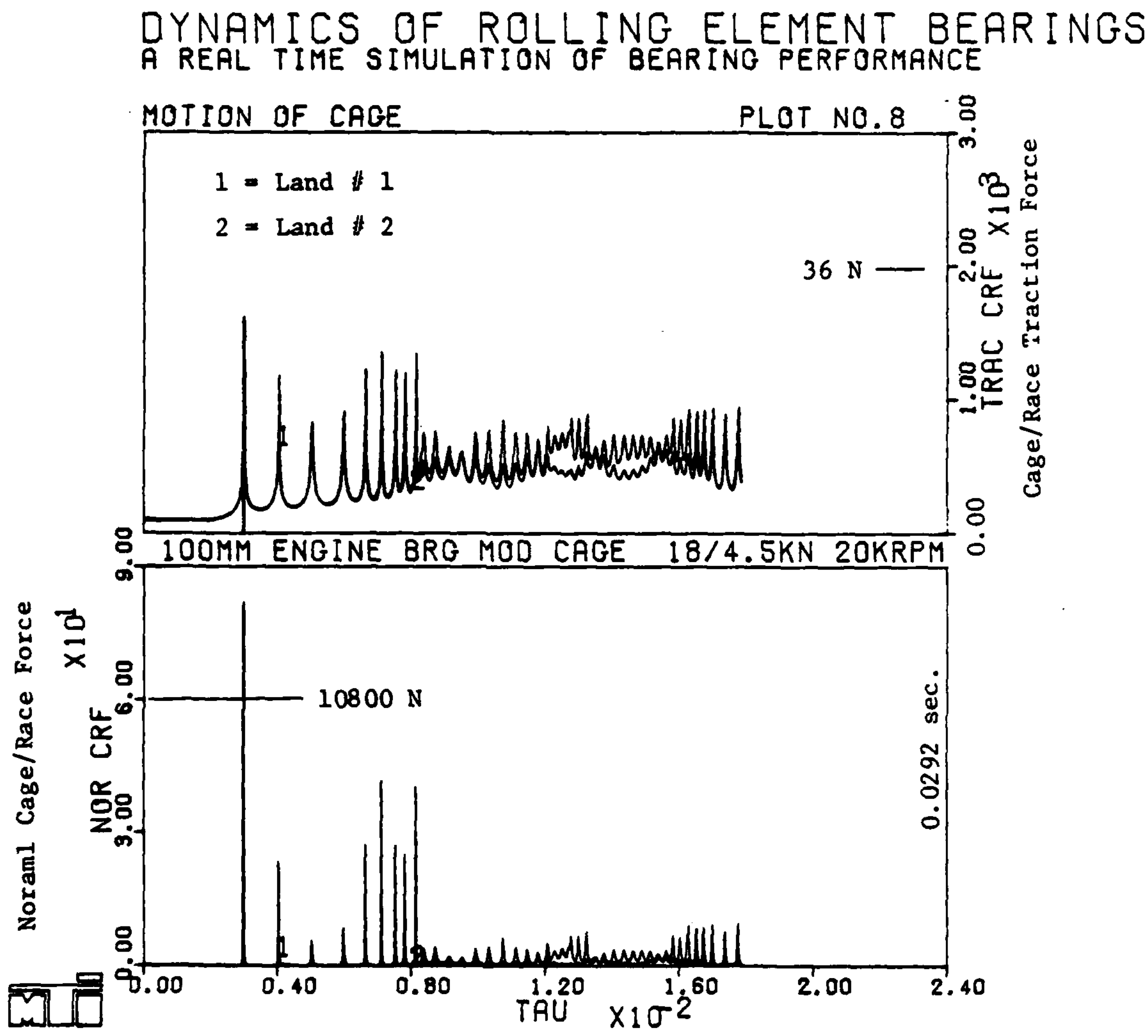

Fig. 5-5 Repeated Cage/Race Collisions in the 100 mm Engine Bearing

force curves. The ultimate behavior to note is that the race/cage collisions are really very close together in time in steady state. Also the magnitude of the force is fairly large ($\sim$ 1800 N). This means that the cage will steadily be in contact with the race with a relatively large force, and some cage wear problems may be expected. This is seen by the steady circular orbit of the cage mass center in Figure 5-6 with the orbit radius equal to the cage/race radial clearance. This effect has indeed been confirmed experimentally [18] and the experimental results will soon be documented elsewhere [19].

Corresponding to the above race/cage collisions considerable radial chatter of the cage mass center is observed, as shown in Figure 5-7 in terms of the cage mass center velocities. Excessive whirl is indicated by the increased orbital velocity. The coning motion is demonstrated by the angular velocity variations in Figure 5-8. The deviation of cage angular velocity from the initial epicyclic angular velocity (see component x) indicates substantial skid in the bearings due to excessive rubbing of the cage at the cage/race interrace. Both cage whirl and skid are better understood in terms of the parameters λ_1 and λ_2 respectively, as shown in Figure 5-9.

Finally, the variations of bearing torque, power loss, and the cumulative load slip integral are shown in Figure 5-10. The initial bump in the torque curves correspond to the gyroscopic slip of the balls as discussed above and the steady noise is a result of the ball/cage and race/cage collisions.

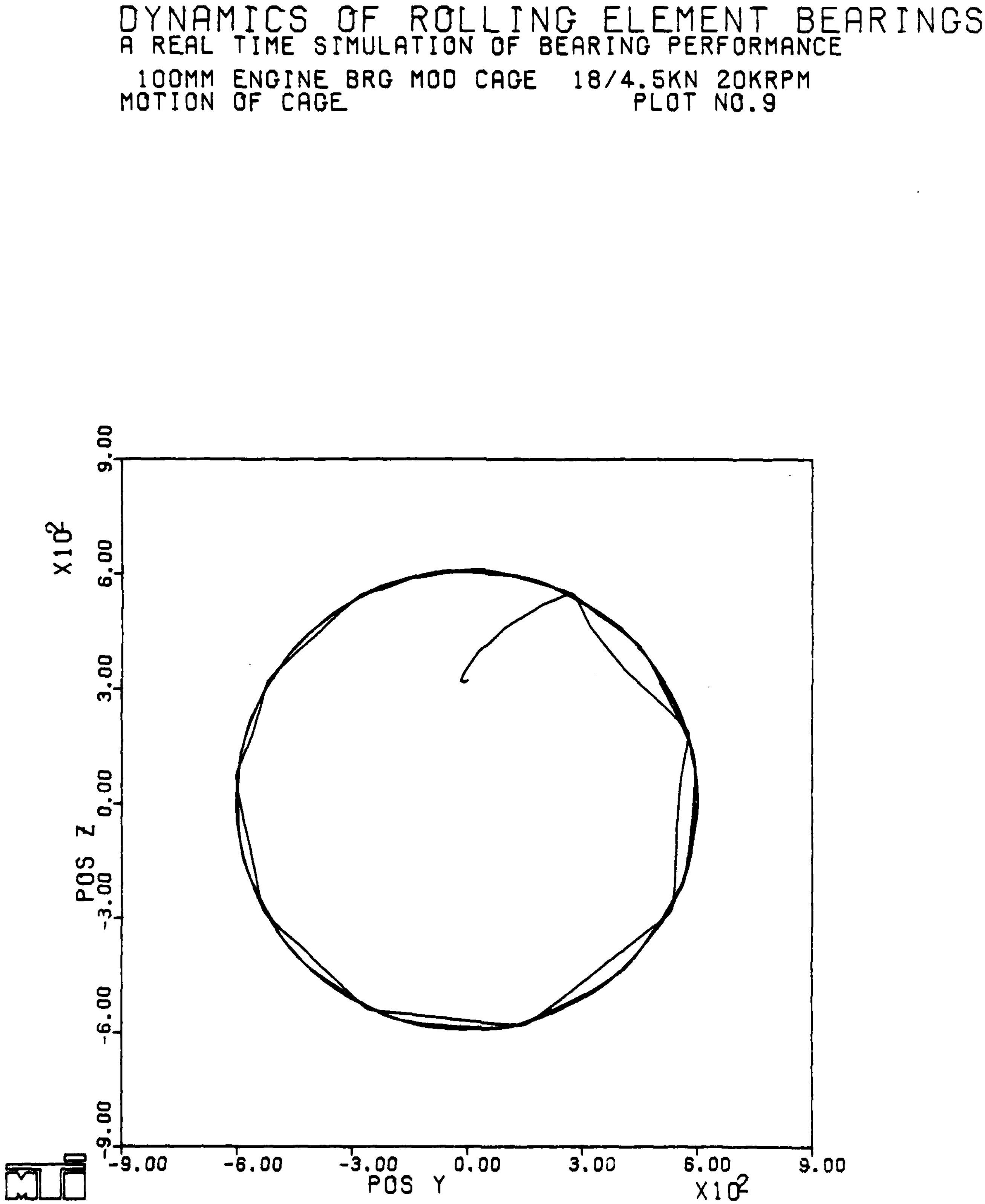

Fig. 5-6 Steady Orbit of the Cage Mass Center for the 100 mm Engine Bearing. Scale = 0.009525 M

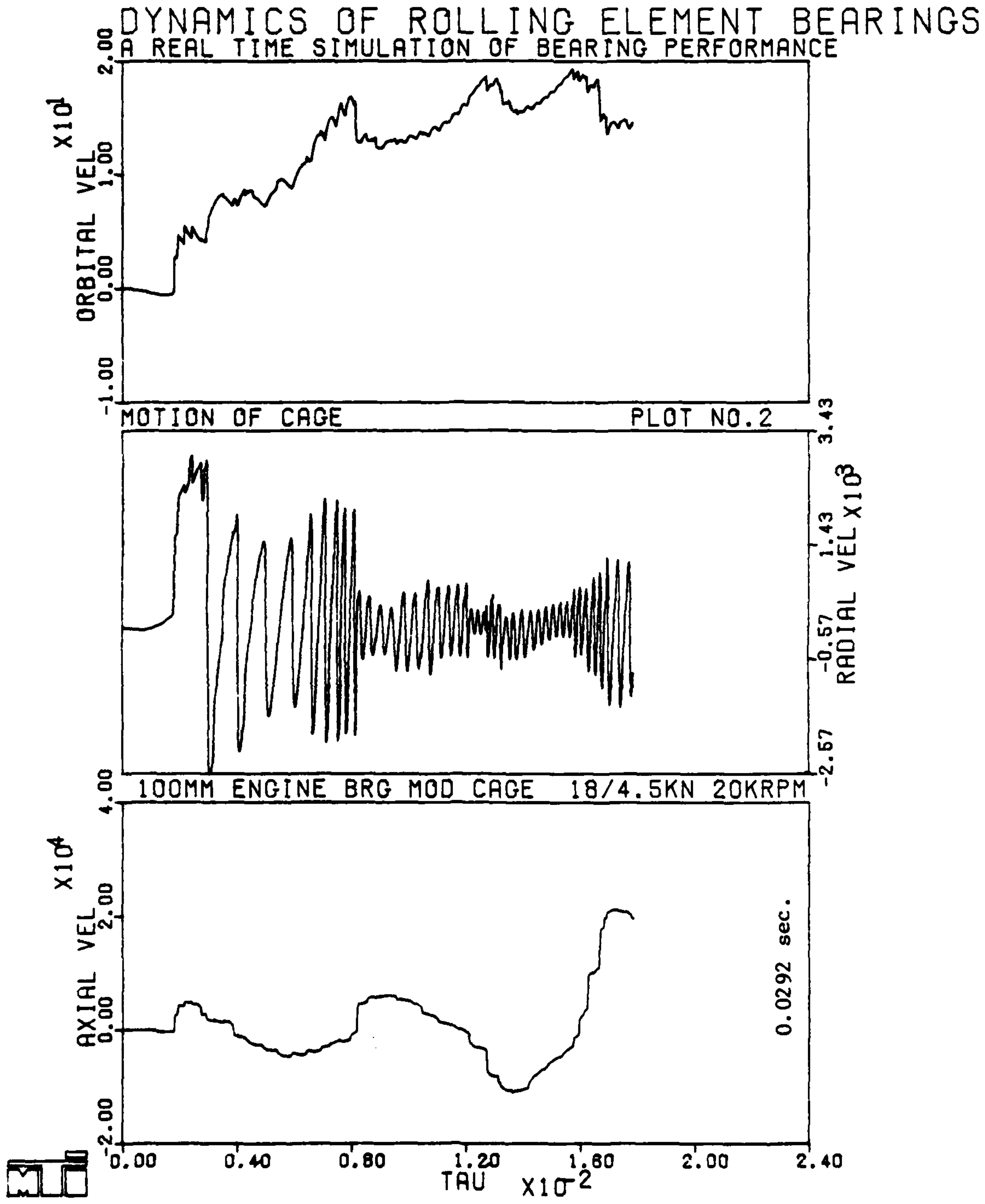

Fig. 5-7 Dimensionless Velocity of Cage Mass Center for the 100 mm Engine Bearing. Scales = 78.17 M/Sec, 7.84 x 10^4 RPM

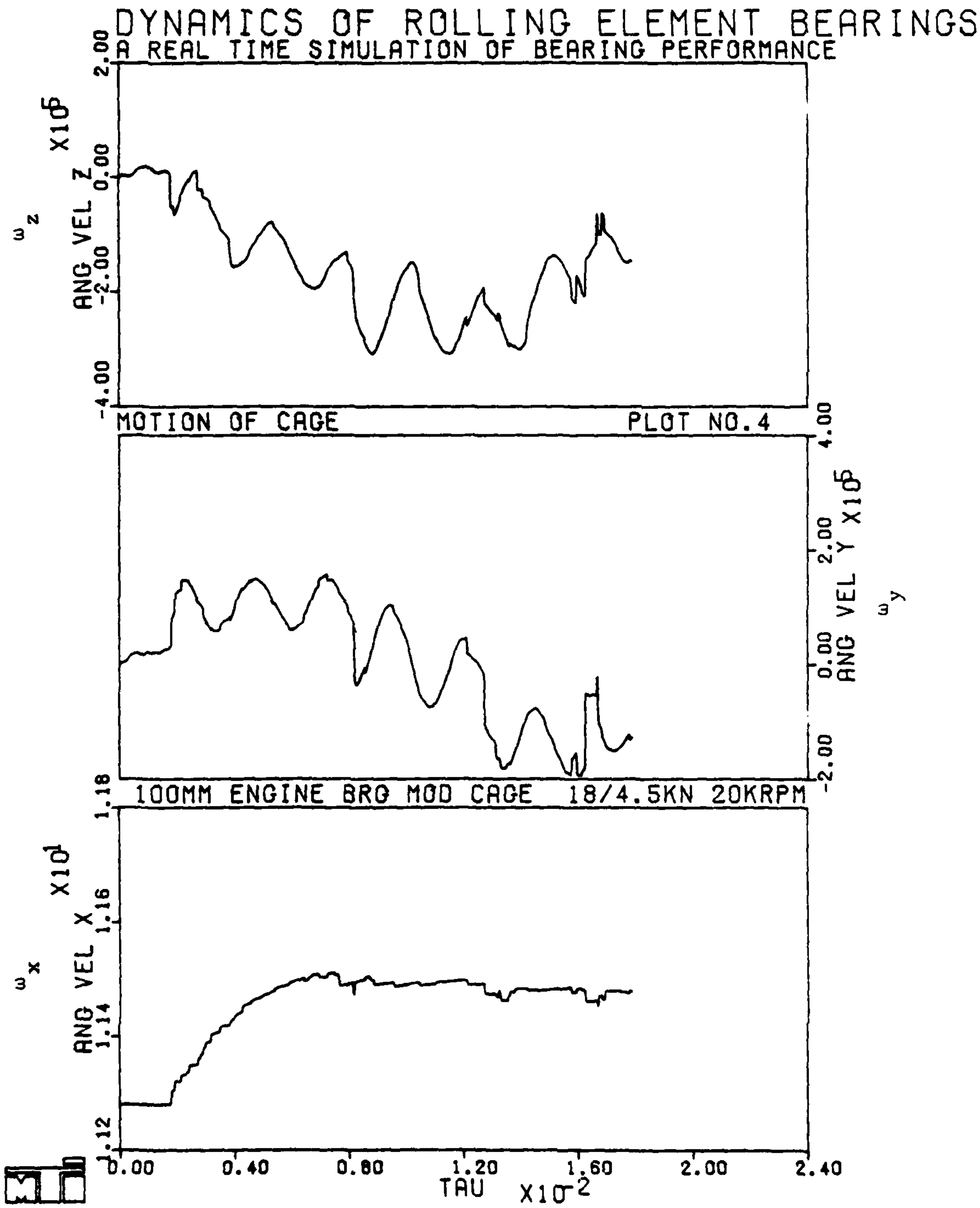

Fig. 5-8 Dimensionless Angular Velocity of a Ball in the 100 mm Engine Bearing. Scale = 7.84×10^4 RPM

DYNAMICS OF ROLLING ELEMENT BEARINGS
A REAL TIME SIMULATION OF BEARING PERFORMANCE

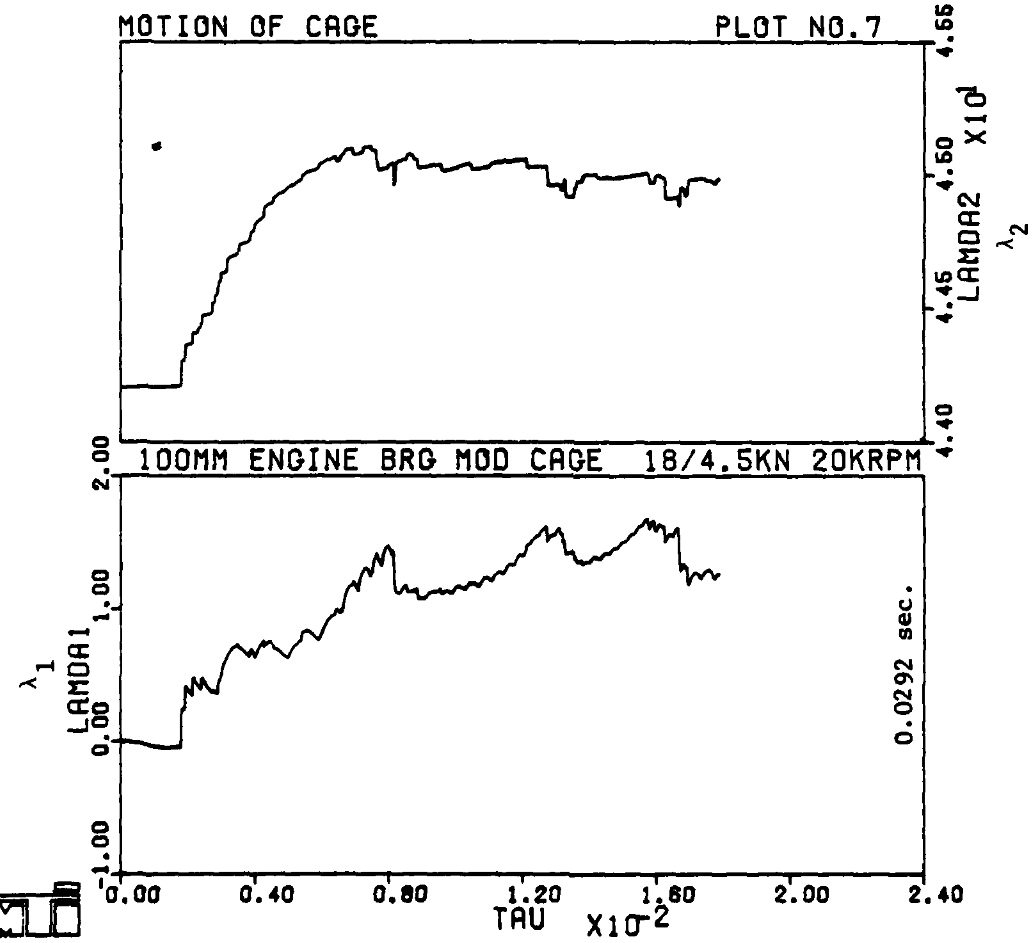

Fig. 5-9 Cage Whirl and Skid Parameters for the 100 mm Engine Bearing

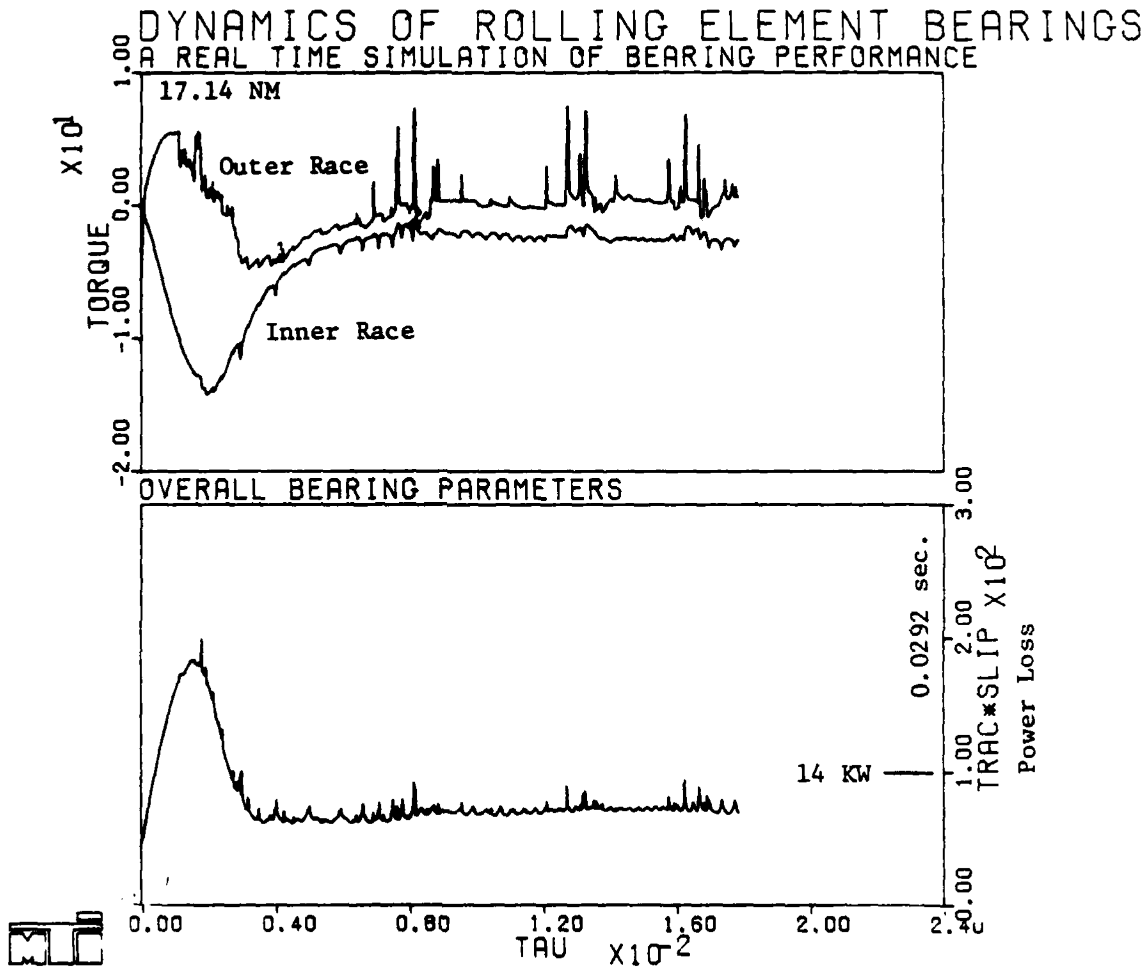

Fig. 5-10 Bearing Torque and Power Loss Variations in the 100 mm Engine Bearing

6.0 CONCLUSIONS AND RECOMMENDATIONS

The existing Dynamics of Rolling Element Bearings (DREB) computer program has been enhanced to selectively suppress the very high frequency vibratory motion of the balls in an angular contact ball bearing and thereby provide the added capability of investigating the low frequency phenomena in some depth. With such an enhancement a considerable increase in the maximum permissible time step size has led to bearing performance simulation over several shaft revolutions within reasonable computing effort. The updated program version is referred to as RAPIDREB. Some of the essential RAPIDREB capabilities include:

1. Radial and axial equilibrium constraints on the balls to eliminate the high frequency ball/race vibration.
2. Introduction of fictitious damping at each contact interface to selectively damp the high frequencies.
3. Incorporation of a number of Runga-Kutta type explicit formulae.
4. A predictor-corrector algorithm for conditions under which no substantial cyclic components and discontinuities in the solutions exist.
5. Option to change the integrating algorithm at any point in time.
6. Treatment of ball guided cages for very low speed ball bearings.
7. Built in options for synchronous radial loading due to unbalance or any rotating loads.

Any of the above options can be selected or the program can be run in a completely generalized mode identical to the original DREB program. It is shown that depending on the operating conditions substantial computing effort can be saved by properly selecting the constraints and the integrating method. By exercising RAPIDREB over a number of trial runs it is concluded that the predictor-corrector type of formula is only suitable when there are no discontinuities and high frequency vibratory motions in the system. Thus after suppressing the very high frequencies, the predictor-corrector schemes offer the benefit of very large step sizes, only for the cases where no discontinuities associated with ball/cage or race/cage collisions are present. This is because extrapolation past a discontinuity results in larger errors in the prediction process. Thus these schemes are useful only for bearings either without cage or with basically hydrodynamic interaction at the ball/cage and race/cage interfaces. Considerable care in selecting the integrating method is therefore necessary.

RAPIDREB is used to produce performance simulations of two very low speed, lightly loaded DMA (Despun Mechanical Assembly) ball bearings and a typical high-load, high-speed ball bearing for engine applications. The DMA bearings have pure thrust load and therefore gravity is the only dominant factor as far as the cage is concerned. Hence both bearings are oriented such that gravity is either parallel or perpendicular to the bearing axis. The first of the DMA bearings is a 100 mm bore ball bearing with inner race guided cage. The steady state performance simulation obtained by integrating the equations of motion over more than a shaft revolution indicate that when gravity acts along the bearing axis the cage contacts all the balls and balances its own weight against the balls. No substantial cage whirl is observed in the simulation and the bearing operation is generally stable. When weight of the cage acts normal to the bearing axis variable ball/cage and cage/race interactions are observed as the balls travel in their orbit. This is basically due to the eccentric location of the cage. An appreciable coning motion of the cage is also observed in this case but again no appreciable cage whirl or skid is observed and the operation is generally stable. The second of the DMA bearings is a 150 mm bore ball bearing with a ball guided cage with a conical guidance surface attached to the nominally cylindrical pocket. Again the first symmetric case when the gravity acts along the bearing axis is similar to the 100 mm bearing and the weight of the cage is balanced against the ball/cage forces. However, large variations in cage whirl velocity are observed indicating some element of instability. The bearing performance worsens as the cage is allowed to move eccentricly under its own weight acting normal to the bearing axis. In this case large interactions at both the cylindrical and conical parts of the cage pocket are observed. This results in a large amount of noise and greatly erratic motion of the cage.

The engine bearing performance simulations are obtained with a combined thrust and radial load (thrust to radial load ratio of 4) over more than seven shaft revolutions. The test example consists of a 100 mm bore ball bearings operating at 20,000 rpm with a thrust load of 18000 N and a radial load of 4500 N. The bearing uses a steel cage guided on the inner race. It is shown that for such an operating environment the cage develops appreciable whirl velocity and in steady state a continued metal contact at the guiding land is established. Hence a definite possibility of cage wear at the guiding surface is predicted.

Since RAPIDREB easily provides bearing performance simulations over several shaft revolutions and the prediction of steady state performance is possible with great confidence, several recommendations for future development may be listed:

1. The steady state performance predictions should be validated experimentally, particularly in terms of the cage motion. This will greatly strengthen the predictive capability of RAPIDREB and thereby enhance its value as a design tool.

2. Since solutions over relatively large time domains are now possible a graphic simulation of the various component motions in the bearing will contribute to greater understanding of the various instabilities and the dynamic behavior of the bearing as a whole. This will essentially require the development of a sophisticated graphics program replacing the current plot program.

3. RAPIDREB can be very useful in establishing direct correlations of bearing life and dynamic stability with both geometrical and operational variables. Hence it can be a very useful tool in guiding accelerated life cycle tests for long life bearings

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INTERACTIVE GRAPHIC SIMULATION OF ROLLING ELEMENT BEARINGS. PHA--ETC(U)
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UNCLASSIFIED MTI-81TR66 AFWAL-TR-81-4148 NL

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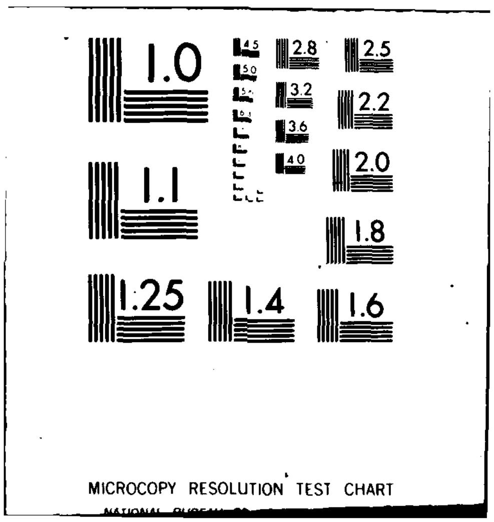
1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART

such as the DMA bearings. A parametric study to establish such correlations and the development of guidelines for accelerated bearing life tests is therefore recommended.

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APPENDIX A

INPUT DATA DESCRIPTION FOR THE COMPUTER PROGRAMS

RDREB

and

RDREBP

```
C
C-----------------------------------------------------------------------
C
C                        PROGRAM  -RDREB-
C                        ----------------
C
C                    RAPID SIMULATION OF THE
C                    =======================
C
C               DYNAMICS OF ROLLING ELEMENT BEARINGS
C               ====================================
C
C               - A REAL TIME PERFORMANCE SIMULATION -
C
C
C                              BY
C
C                        PRADEEP K. GUPTA
C               MECHANICAL TECHNOLOGY INCORPORATED
C                    968 ALBANY-SHAKER ROAD
C                    LATHAM, NEW YORK 12110
C                           U. S. A.
C
C                         SPONSORED BY
C               WRIGHT-PATTERSON AIR FORCE BASE
C                             OHIO
C                  CONTRACT NO. F33615-80-C-5152
C
C
C   FOR THE PERFORMANCE SIMULATION OF BALL BEARINGS, THIS PROGRAM
C   IS AN ADVANCED VERSION OF THE BALL BEARING TREATMENT IN THE MTI
C   DYNAMICS OF ROLLING ELEMENT BEARING (DREB) COMPUTER PROGRAM.  THE
C   ADDED CAPABILITIES INCLUDE --
C
C       1.  SELECTIVE CONSTRAINTS OF THE AXIAL AND RADIAL MOTION OF
C           THE BALLS AND THEREBY ELIMINATING HIGH FREQUENCY BALL/RACE
C           VIBRATIONS.
C
C       2.  DAMPING COEFFICIENTS AT ALL INTERACTIONS WHICH ALLOW FOR
C           EITHER A REALISTIC INTERNAL MATERIAL DAMPING OR A
C           FICTITIOUS DAMPING TO SELECTIVELY DAMP OUT CERTAIN COMPO-
C           NENTS OF THE VIBRATORY MOTION.
C
C       3.  A NUMBER OF DIFFERENT INTEGRATING ALGORITHMS TO INTEGRATE
C           THE DIFFERENTIAL EQUATIONS OF MOTION.
C
C       4.  A TREATMENT FOR A BALL GUIDED CAGE.
C
C       5.  IMPROVED BALL/RACE TRACTION INTEGRAL SPECIALLY FOR
C           SOLID LUBRICATED CONDITIONS.
C
C       6.  SWITCH FOR SIMULATING A ROTATING RADIAL LOAD ON THE
C           BEARING.
```

```
C
C       7.  IMPROVED COMPUTING SPEEDS.  PROPER SELECTION OF CONS-
C           TRAINTS AND INTEGRATING METHOD CAN EASILY REDUCE THE COM-
C           PUTATION EFFORT BY A FACTOR OF FIFTEEN TO TWENTY WHEN
C           COMPARED WITH THE ORIGINAL DREB PROGRAM.
C
C     THE PROGRAM -RDREB- IS DESIGNED FOR BALL BEARINGS ONLY AND NO
C     TREATMENT OF ROLLER BEARINGS IS AVAILABLE.
C
C
C     INPUT DATA
C     ==========
C
C     ALL INPUT DATA IS SUPPLIED ON DATA CARDS IN COLUMNS 11 TO 70.
C     THE FIRST AND THE LAST TEN COLUMNS ON EACH CARD ARE RESERVED FOR
C     IDENTIFICATION PURPOSES.  THE USER MAY USE THESE COLUMNS FOR ANY
C     DESIRABLE INFORMATION.  THE INFORMATION IS JUST READ IN AND
C     PRINTED OUT IN THE LIST OF DATA CARDS.
C
C     THE PROGRAM ALLOWS FOR BOTH THE ENGLISH AND SI SYSTEMS OF UNITS.
C     ALL INPUT DATA MUST BE IN THE SYSTEM OF UNIT SELECTED BY THE USER.
C     THE FUNDAMENTAL UNITS AND THE ABBREVIATIONS USED IN THE TWO
C     SYSTEMS ARE DESCRIBED IN THE FOLLOWING TABLE --
C
C       QUANTITY        ENGLISH UNIT             SI UNIT
C
C       FORCE  (F)      POUND FORCE  (LBF)       NEWTON  (N)
C       LENGTH (L)      INCH (IN)                METER  (M)
C       TIME  (T)       SECOND  (S)              SECOND  (S)
C       MASS  (MA)      POUND MASS  (LBM)        KILOGRAM MASS  (KGM)
C       TEMP  (TM)      DEG RANKINE  (DEG-R)     DEG KELVIN  (DEG-K)
C
C     APPROPRIATE UNITS FOR ALL THE PHYSICAL QUANTITIES CAN BE DERIVED
C     IN TERMS OF THE ABOVE FUNDAMENTAL UNITS.  SIMPLY FOR CONVENIENCE
C     ALL ANGULAR VELOCITIES AND ANGULAR ACCELERATIONS ARE SPECIFIED IN
C     RPM AND RPM/SEC RESPECTIVELY. ALSO, ALL ANGLES ARE SPECIFIED IN
C     DEGREES.
C
C     ALL INPUT VARIABLE NAMES BEGINNING WITH I, J, K, L, M AND N ARE
C     INTEGERS AND THE REMAINING ARE REAL NUMBERS.  ALSO, IT WILL BE
C     IMPORTANT THAT ALL INTEGERS ARE RIGHT ADJUSTED IN THEIR FIELDS.
C
C     AN ABBREVIATION -RE- IS USED FOR ROLLING ELEMENT IN THE FOLLOWING
C     INPUT DESCRIPTION AND ALSO IN THE PRINT OUTPUT OF THE PROGRAM.
C
C     THE VARIABLE NAMES, COLUMN FIELDS AND VARIABLE DESCRIPTIONS ARE
C     DOCUMENTED BELOW FOR EACH INPUT DATA CARD.
C
C     CARD 1
C     ------
C       MODE    11-12   SWITCH WITH FOLLOWING FUNCTIONS --
C                          DYNAMIC SOLUTION WITH NO
C                          CHANGE IN OPERATING DATA    = 0
C                          DYNAMIC SOLUTION WITH
C                          MODIFIED OPERATING DATA ON
C                          CARDS 13-19                 = 1
C                          DYNAMIC SOLUTION WITH ARBI-
```

```
C                          TRARY INITIAL CONDITIONS    = 2              RDREB116
C                          AXIAL AND RADIAL BALL MOTION                 RDREB117
C                          CONSTRAINED BY QUASI-STATIC                  RDREB118
C                          EQUILIBRIUM AT EACH STEP    = 3              RDREB119
C                          CONSTRAINED AXIAL AND RADIAL                 RDREB120
C                          BALL MOTION UNDER PURE                       RDREB121
C                          THRUST LOAD AND CONSTANT                     RDREB122
C                          RACE SPEEDS                 = 4              RDREB123
C    IR     13-16       RECORD NUMBER FOR INITIAL CONDITIONS. IR=0 AT   RDREB124
C                       START.                                          RDREB125
C    MONT    17-19      DATA MONITOR CODE. WHEN MODULUS OF (IQ-1), IQ   RDREB126
C                       BEING THE STEP NUMBER, AND MONT VANISHES.  THE  RDREB127
C                       DIMENSIONLESS SOLUTION VECTOR WILL BE WRITTEN   RDREB128
C                       IN THE MASTER DATA FILE.                        RDREB129
C    NPRT    20-28      VECTOR OF LENGTH 3 CONTAINING THE PRINT CODES   RDREB130
C                       WITH THE FOLLOWING SIGNIFICANCE --              RDREB131
C                       NPRT(1) = 0, PRINT ALL ROLLING ELEMENT SOLUTIONS  RDREB132
C                                 N, SKIP N ROLLING ELEMENT SOLUTIONS   RDREB133
C                                -N, SKIP ALL ROLLING ELEMENT SOLUTIONS  RDREB134
C                       NPRT(2)=K SUCH THAT WHEN THE MODULUS OF (IQ-1),  RDREB135
C                                 IQ BEING THE STEP NUMBER, AND K       RDREB136
C                                 VANISHES, THE OUTPUT IS PRINTED AT    RDREB137
C                                 STEP IQ.                              RDREB138
C                       NPRT(3)=K SUCH THAT WHEN THE MODULUS OF (IOP-1),  RDREB139
C                                 IOP BEING THE STEP NUMBER AT WHICH OUT-  RDREB140
C                                 PUT IS PRINTED, AND K VANISHES, THE   RDREB141
C                                 DIMENSIONLESS SOLUTION VECTOR IS ALSO  RDREB142
C                                 PRINTED.                              RDREB143
C                       EACH COMPONENT HAS A FIELD WIDTH OF 3.          RDREB144
C    NOPT    29-40      VECTOR OF LENGTH 3 CONTAINING THE OPTIMIZATION  RDREB145
C                       CODES (SEE NOTE 5) DEFINED AS --                RDREB146
C                       NOPT(1) = NUMBER OF STEPS FOR WHICH INTEGRATION  RDREB147
C                                 WILL CONTINUE WHEN THE TRUNCATION ERROR  RDREB148
C                                 EXCEEDS THE SPECIFIED LIMIT WITH THE  RDREB149
C                                 MINIMUM STEP SIZE.                    RDREB150
C                       NOPT(2) = NUMBER OF STEPS AFTER WHICH AN ATTEMPT  RDREB151
C                                 TO OPTIMIZE THE STEP SIZE WILL BE MADE.  RDREB152
C                       NOPT(3) = MAXIMUM NUMBER OF STEPS IN THE RUN.   RDREB153
C                       EACH COMPONENT HAS A FIELD WIDTH OF 4.          RDREB154
C    IMET    41-43      SWITH FOR INTEGRATING ALGORITHM --              RDREB155
C                       IMET =  0, FOURTH ORDER RUNGA-KUTTA-MERSON      RDREB156
C                               1, FOURTH ORDER RUNGA-KUTTA-FEHLBERG    RDREB157
C                               2, FOURTH ORDER RUNGA-KUTTA-ENGLAND     RDREB158
C                               3, CLASSICAL FOURTH ORDER RUNGA-KUTTA   RDREB159
C                               4, FOURTH ORDER RALSTON-RUNGA-KUTTA     RDREB160
C                               5, FIFTH ORDER KUTTA-NYSTROM            RDREB161
C                               6, SIXTH ORDER HUTTA-RUNGA-KUTTA        RDREB162
C                               7, SECOND ORDER RUNGA-KUTTA             RDREB163
C                               8, THIRD ORDER RUNGA-KUTTA              RDREB164
C                              10, A PREDICTOR-CORRECTOR FORMULA        RDREB165
C                                  (IMET=10 RECOMMENDED FOR CAGELESS    RDREB166
C                                  BEARINGS ONLY)                       RDREB167
C                                                                       RDREB168
C   CARD 2                                                              RDREB169
C   ------                                                              RDREB170
C                                                                       RDREB171
C   SEE NOTE 6 FOR DISCUSSION RELEVANT TO THIS CARD.                    RDREB172
```

```
C
C     DINT    11-22    INITIAL DIMENSIONLESS TIME STEP SIZE.
C     DMIN    23-34    MINIMUM DIMENSIONLESS TIME STEP SIZE.
C     DMAX    35-46    MAXIMUM DIMENSIONLESS TIME STEP SIZE.
C       TF    47-58    FINAL DIMENSIONLESS TIME.
C
C *************************************************************
C *  IF IR.GT.0 AND MODE.EQ.0 GO TO CARD 21.                  *
C *  IF IR.GT.0 AND MODE.EQ.1 GO TO CARD 13.1.                *
C *************************************************************
C
C    CARD 3
C    ------
C      KBT    11-13    BEARING TYPE DEFINED AS FOLLOWS --
C                      BEARING WITH CAGE = 1
C                      CAGELESS BEARING = -1
C      JNT    14-16    SWITCH FOR TRACTION INTEGRAL --
C                         SINGLE INTEGRAL FOR EHD
C                         CONDITIONS                  = 0
C                         DOUBLE INTEGRAL FOR SOLID
C                         LUB CONDITIONS              = 1
C      MRR    17-19    SWITCH FOR RE AND RACE MATERIALS --
C                         STANDARD DATA               = 0
C                         DATA SUPPLIED ON CARDS
C                         10.1 AND 10.2               = 1
C      MHS    20-22    SWITCH FOR HOUSING AND SHAFT MATERIALS --
C                         STANDARD DATA               = 0
C                         DATA SUPPLIED ON CARDS
C                         10.3 AND 10.4               = 1
C    ISPEC    23-70    BEARING SPECIFICATION CODE.  ANY ALPHANUMERIC
C                      STRING (A MAXIMUM OF 48 CHARACTERS) BY WHICH ALL
C                      PRINT AND PLOT OUTPUTS WILL BE IDENTIFIED.
C
C    CARD 4
C    ------
C      BOR    11-22    BORE OR SHAFT OUTER DIA (L).
C       OD    23-34    OUTER RACE OUTER DIA (L).
C      SDA    35-46    SHAFT INNER DIA (L).
C      HDA    47-58    HOUSING OUTER DIA (L).
C
C    CARD 5
C    ------
C       SF    11-34    VECTOR OF LENGTH 2 CONTAINING THE DIAMETRAL
C                      INTERFERENCE (L) AT THE OUTER RACE/HOUSING AND
C                      SHAFT/INNER RACE FITS RESPLY.
C                      EACH COMPONENT HAS A FIELD WIDTH OF 12.
C
C    CARD 6
C    ------
C      KTC    11-16    TRACTION CODE DEFINED AS FOLLOWS --
C                         SIMULATED TRACTION CURVE    = 0
C                         SHELL TURBO-33 OIL          = 1
C                         5P4E POLYPHENOL ETHER       = 2
C                         MIL-L-7808 OIL              = 3
C                         ANY ARBITRARY MODEL         = 4
C       NR    17-22    NUMBER OF ROLLING ELEMENTS.
C       LU    23-28    SWITCH FOR SELECTION OF UNITS --
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C                         IN-LB UNITS                   = 1
C                         SI UNITS                      = 2
C        NBW     29-34    NUMBER OF ELEMENTS FOR WHICH OUTPUT WILL BE
C                         MONITORED.  UPPER LIMIT IS SIX (6).
C        IBW     35-70    VECTOR OF LENGTH NBW.  THE COMPONENTS WILL DENOTE
C                         THE INDICES OF THE BEARING ELEMENTS.
C                         EACH COMPONENT HAS A FIELD WIDTH OF 6.
C
C     CARD 7
C     ------
C
C         DF     11-22    BALL DIAMETER (L).
C         DP     23-34    PITCH DIAMETER (L).
C       ALFA     35-46    CONTACT ANGLE (DEG) OR DIAMETRAL PLAY (L).
C                         A POSITIVE VALUE DENOTES CONTACT ANGLE AND A
C                         NEGATIVE VALUE DENOTES A DIAMETRAL PLAY EQUAL TO
C                         THE ABSOLUTE VALUE OF ALFA.
C     RCUR(1)    47-58    OUTER RACE CUR FACTOR.
C     RCUR(2)    59-70    INNER RACE CUR FACTOR.
C
C *****************************************************************
C *   THE SERIES 8 CARDS ARE REQUIRED ONLY IN CASE OF DYNAMIC     *
C *   SOLUTIONS.  MODE.GT.0 ON CARD 1.                            *
C *****************************************************************
C
C     CARD 8.1
C     --------
C
C     THIS CARD CONTAINS ROLLING ELEMENT INERTIAL PARAMETERS.  IF
C     STANDARD VALUES ARE DESIRED THEN SET ALL VALUES EQUAL TO ZERO.
C
C        XMB     11-22    RE MASS (MA).
C        XIX     23-34    RE POLAR MOMENT OF INERTIA (MA*L**3).
C        XIY     35-46    RE TRANSVERSE MOMENT OF INERTIA (MA*L**3).
C
C     CARD 8.2
C     --------
C
C     THIS CARD CONTAINS CAGE INERTIAL PARAMETERS.  FOR STANDARD VALUES
C     OR FOR CAGELESS BEARING SET ALL VALUES EQUAL TO ZERO.
C
C        XMC     11-22    CAGE MASS (MA).
C        XKX     23-34    CAGE POLAR MOMENT OF INERTIA (MA*L**3).
C        XKY     35-46    CAGE TRANSVERSE MOMENT OF INERDIA (MA*L**3).
C
C     CARD 8.3 AND 8.4
C     ----------------
C
C     THE FOLLOWING TWO CARDS (I=1,2) WILL CONTAIN THE EFFECTIVE
C     INERTIAL PARAMETERS FOR THE OUTER (I=1) AND INNER (I=2) RACES
C     RESPLY.  IF THE RACE IS CONSTRAINED IN ACCORDANCE WITH THE
C     SPECIFIED MOTION THEN THE RELEVANT PARAMETERS MAY BE SET TO ZERO.
C     ALSO, FOR STANDARD VALUES, SET ALL PARAMETERS EQUAL TO ZERO.
C
C     XMR(I)     11-22    RACE MASS (MA).
C     XJX(I)     23-34    RACE POLAR MOMENT OF INERTIA (MA*L**3).
C     XJY(I)     35-46    RACE TRANSVERSE MOMENT OF INERTIA (MA*L**3).
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C                                                                       RDREB287
C     CARD 8.5                                                          RDREB288
C     --------                                                          RDREB289
C       BRDP    11-22   BALL/RACE DAMPING RATIO                         RDREB290
C       BCDP    23-34   BALL/CAGE DAMPING RATIO                         RDREB291
C       RCDP    35-46   CAGE/RACE DAMPING RATIO                         RDREB292
C                                                                       RDREB293
C ********************************************************************* RDREB294
C *   THE SERIES 9 CARDS ARE REQUIRED ONLY WHEN A CAGE IS PRESENT,     * RDREB295
C *   (KBT.GT.0).  IF NO CAGE EXISTS GO TO CARD 10.1                   * RDREB296
C ********************************************************************* RDREB297
C                                                                       RDREB298
C     CARD 9.1                                                          RDREB299
C     --------                                                          RDREB300
C        MCR    11-16   SWITCH FOR CAGE MATERIALS --                    RDREB301
C                          STANDARD DATA                  = 0           RDREB302
C                          DATA SUPPLIED ON CARD 10.5     = 1           RDREB303
C       IHBC    17-22   SWITCH FOR RE/CAGE CONTACT --                   RDREB304
C                          HYDRODYNAMICS NEGLECTED        = 0           RDREB305
C                          HYDRODYNAMICS INCLUDED         = 1           RDREB306
C       IHRC    23-28   SWITCH FOR CAGE/RACE CONTACT --                 RDREB307
C                          HYDRODYNAMICS NEGLECTED        = 0           RDREB308
C                          HYDRODYNAMICS INCLUDED BUT                   RDREB309
C                          NO SQUEEZE FILM DAMPING        = 1           RDREB310
C                          HYDRODYNAMICS INCLUDED                       RDREB311
C                          WITH SQUEEZE FILM DAMPING      = 2           RDREB312
C        IBC    29-34   SWITCH FOR RE/CAGE CONTACT --                   RDREB313
C                          USE HERTZIAN SPRING            = 0           RDREB314
C                          SPRING SUPPLIED IN -PSPRG-     = 1           RDREB315
C        IRC    35-40   SWITCH FOR CAGE/RACE CONTACT --                 RDREB316
C                          USE HERTZIAN SPRING            = 0           RDREB317
C                          SPRING SUPPLIED IN -RSPRG-     = 1           RDREB318
C       IRGS    41-52   VECTOR OF LENGTH 2.  COMPONENTS INDICATE TYPE OF RDREB319
C                       GUIDANCE FOR THE TWO LANDS ALONG THE POSITIVE   RDREB320
C                       AND NEGATIVE X-AXIS RESPLY --                   RDREB321
C                          OUTER RACE GUIDANCE            = 1           RDREB322
C                          INNER RACE GUIDANCE            = 2           RDREB323
C                       EACH COMPONENT HAS A FIELD WIDTH OF 6.          RDREB324
C                                                                       RDREB325
C     CARD 9.2                                                          RDREB326
C     --------                                                          RDREB327
C       CDIA    11-34   VECTOR OF LENGTH 2 CONTAINING THE EFFECTIVE     RDREB328
C                       OUTER AND INNER DIAMETERS (L) OF CAGE.          RDREB329
C                       EACH COMPONENT HAS A FIELD WIDTH OF 12.         RDREB330
C        CWC    35-46   EFFECTIVE CAGE WIDTH (L) FOR MASS COMPUTATION.  RDREB331
C        CCL    47-70   VECTOR OF LENGTH 2 CONTAINING THE EFFECTIVE     RDREB332
C                       DIAMETRAL CLEARANCES (L) AT THE CAGE/OUTER RACE RDREB333
C                       AND CAGE/INNER RACE INTERACTIONS RESPLY FOR THE RDREB334
C                       COMPUTATION OF CHURNING EFFECTS.                RDREB335
C                       EACH COMPONENT HAS A FIELD WIDTH OF 12.         RDREB336
C                                                                       RDREB337
C     CARDS 9.3.1 AND 9.3.2                                             RDREB338
C     ---------------------                                             RDREB339
C                                                                       RDREB340
C ********************************************************************* RDREB341
C *   CARD 9.3.1 IS ONLY REQUIRED IF IRGS(1).NE.0                      * RDREB342
C ********************************************************************* RDREB343
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C                                                                       RDREB344
C     THESE TWO CARDS (I=1,2) CONTAIN DATA FOR THE TWO GUIDING LANDS    RDREB345
C     ALONG THE POSITIVE (I=1) AND NEGATIVE (I=2) X-AXIS RESPLY.        RDREB346
C                                                                       RDREB347
C     RGL(I)    11-22   GUIDING RACE RADIUS (L).                        RDREB348
C     CRL(I)    23-34   GUIDING CAGE RADIUS (L).                        RDREB349
C     CWS(I)    35-46   LAND WIDTH (L)                                  RDREB350
C     CLS(I)    47-58   DIS OF LAND OUTER EDGE FROM CAGE CENTER (L).    RDREB351
C                                                                       RDREB352
C     CARD 9.4                                                          RDREB353
C     --------                                                          RDREB354
C        RCL    11-22   RE/CAGE DIAMETRAL CLEARANCE (L).                RDREB355
C       RCLH    23-34   RE/CAGE MAX LUB FILM (L) .LT. RCL FOR STARVED   RDREB356
C                       CONTACT.  RCLH=RCL FOR FULLY FLOODED CONDITIONS. RDREB357
C                       THIS INPUT IS RELEVANT ONLY FOR BALL BEARINGS.  RDREB358
C       PAN1    35-46   CONE ANGLE (DEG) FOR RE/CAGE GUIDE LAND.        RDREB359
C                       = 0. FOR NO RE/CAGE GUIDANCE.                   RDREB360
C       PAN2    47-58   CONE HEIGHT (L).                                RDREB361
C                       .GT. 0. FOR GUIDANCE CONE ON OUTER DIAMETER.    RDREB362
C                       .LT. 0. FOR GUIDANCE CONE ON INNER DIAMETER.    RDREB363
C                                                                       RDREB364
C     CARD 9.5                                                          RDREB365
C     --------                                                          RDREB366
C       XPOS    11-22   INITIAL AXIAL POSITION (L) OF CAGE MASS CENTER  RDREB367
C                       RELATIVE TO THE MEAN AXIAL POSITION OF ROLLING  RDREB368
C                       ELEMENTS.                                       RDREB369
C       RPOS    23-34   INITIAL RADIAL POSITION (L) OF CAGE MASS CENTER RDREB370
C                       RELATIVE TO THE CENTER OF THE GUIDING RACEWAY   RDREB371
C                       (BEARING CENTER IN CASE OF RE GUIDANCE)         RDREB372
C       APOS    35-46   INITIAL ROTATION (DEG) OF CAGE ABOUT THE X-AXIS. RDREB373
C                                                                       RDREB374
C     CARD 10.1                                                         RDREB375
C     ---------                                                         RDREB376
C                                                                       RDREB377
C ********************************************************************* RDREB378
C *   THIS CARD IS REQUIRED ONLY IF MBP=1 ON CARD 3.                  * RDREB379
C ********************************************************************* RDREB380
C                                                                       RDREB381
C         EB    11-22   RE ELASTIC MODULUS (F/L**2).                    RDREB382
C        POB    23-34   RE POISSON-S RATIO.                             RDREB383
C        DEN    35-46   RE MATERIAL DENSITY (MA/L**3).                  RDREB384
C                                                                       RDREB385
C     CARD 10.2                                                         RDREB386
C     ---------                                                         RDREB387
C                                                                       RDREB388
C ********************************************************************* RDREB389
C *   THIS CARD IS REQUIRED ONLY IF MBP=1 ON CARD 3.                  * RDREB390
C ********************************************************************* RDREB391
C                                                                       RDREB392
C         ER    11-22   RACE ELASTIC MODULUS (F/L**2).                  RDREB393
C        POR    23-34   RACE POISSON-S RATIO.                           RDREB394
C       DENR    35-46   RACE MATERIAL DENSITY (MA/L**3).                RDREB395
C                                                                       RDREB396
C     CARD 10.3                                                         RDREB397
C     --------                                                          RDREB398
C                                                                       RDREB399
C ********************************************************************* RDREB400
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```
C *  THIS CARD IS REQUIRED ONLY IF MHS=1 ON CARD 3.                      *
C ***********************************************************************
C
C        ES    11-22   SHAFT ELASTIC MODULUS (F/L**2).
C       POS    23-34   SHAFT POISSON-S RATIO.
C      DENS    35-46   SHAFT MATERIAL DENSITY (MA/L**3).
C
C    CARD 10.4
C    ---------
C
C ***********************************************************************
C *  THIS CARD IS REQUIRED ONLY IF MHS=1 ON CARD 3.                      *
C ***********************************************************************
C
C        EH    11-22   HOUSING ELASTIC MODULUS (F/L**2).
C       POH    23-34   HOUSING POISSON-S RATIO.
C      DENH    35-46   HOUSING MATERIAL DENSITY (MA/L**3).
C
C    CARD 10.5
C    ---------
C
C ***********************************************************************
C *  THIS CARD IS REQUIRED ONLY IF MCR=1 ON CARD 9.1.                    *
C ***********************************************************************
C
C        EC    11-22   CAGE ELASTIC MODULUS (F/L**2).
C       POC    23-34   CAGE POISSON-S RATIO.
C      DENC    35-46   CAGE MATERIAL DENSITY (MA/L**3).
C
C    CARD 11
C    -------
C        JF    11-16   VECTOR OF LENGTH 2.  COMPONENTS DENOTE INITIAL
C                      RACE CONSTRAINTS ALONG AXIAL AND RADIAL
C                      DIRECTIONS (FOR QUASI-STATIC COMPUTATIONS) --
C                         SPECIFIED FORCE            = 0
C                         SPECIFIED DISPLACEMENT     = 1
C                      EACH COMPONENT HAS A FIELD WIDTH OF 3.
C        JN    17-19   NUMBER OF ROLLING ELEMENTS LAGGING THE ZERO DEG
C                      POSITION IN THE CASE OF A PARTIAL BEARING.
C      IROT    20-22   SWITCH FOR ROTATING LOAD --
C                         STATIONARY RADIAL LOAD      = 0
C                         ROTATING RADIAL LOAD WITH
C                         INNER RACE ROTATION          = 1
C        QF    23-46   VECTOR OF LENGTH 2. COMPONENTS DENOTE EITHER
C                      FORCE (F) OR DISPLACEMENT (L) ALONG THE AXIAL AND
C                      RADIAL DIRECTIONS, AS PER CONSTRAINTS SPECIFIED
C                      BY VECTOR JF.
C                      EACH COMPONENT HAS A FIELD WIDTH OF 12.
C      GAMA    47-58   INITIAL MISALIGNMENT (DEG)
C       ARC    59-70   ANGLE (DEG) OVER WHICH NO RE EXISTS IN THE CASE
C                      OF PARTIAL BEARING.  ARC=0. FOR FULL BEARING.
C
C    CARD 12
C    -------
C       RPM    11-34   VECTOR OF LENGTH 2.  THE COMPONENTS CONTAIN THE
C                      INITIAL ANGULAR VELOCITIES (RPM) OF THE OUTER
C                      AND INNER RACES RESPLY.
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C                      EACH COMPONENT HAS A FIELD WIDTH OF 12.
C
C     CARD 13.1
C     ---------
C       FCF1    11-58  VECTOR OF LENGTH 4.  THE COMPONENTS DENOTE
C                      TRACTION PARAMETERS WHEN KTC=0, ON CARD 6, OR
C                      WHEN THE ACTUAL CONDITIONS ARE OUT OF LUB MODEL
C                      BOUNDS.  THESE PARAMETERS ARE ALSO USED FOR THE
C                      ROLLER/FLANGE INTERACTIONS IN CASE OF A ROLLER
C                      BEARING.  THE FOUR COMPONENTS ARE DEFINED AS --
C                         (1) TRACTION COEFF AT ZERO SLIP.
C                         (2) MAX TRACTION COEFF.
C                         (3) TRACTION COEFF AT INFINITE SLIP.
C                         (4) SLIP (L/T) AT MAX TRACTION COEFF.
C                      THE TRACTION COEFF, MU, IS RELATED TO THE SLIP
C                      VELOCITY, V, BY THE RELATION OF THE FORM --
C                         MU = (A+B*V)*EXP(C*V)+D
C                      WHERE THE COEFFICIENTS A, B, C AND D ARE DETER-
C                      MINED BY THE ABOVE FOUR CONDITIONS.  WHEN THE
C                      FOURTH COMPONENT OF FCF1 IS SET EQUAL TO ZERO,
C                      IT IS USED AS A SWITCH AND THE FIRST THREE
C                      COMPONENTS MUST THEN CONTAIN THE VALUES OF A, B
C                      AND C DIRECTLY.  IT SHOULD BE NOTED THAT B AND C
C                      HAVE DIMENSIONS (T/L).  IF THE FOURTH COMPONENT
C                      IS NEGATIVE THEN THE VALUES FOR A, B AND C ARE
C                      STILL CONTAINED IN THE FIRST THREE COMPONENTS,
C                      HOWEVER, THE MAXIMUM VALUE OF THE TRACTION
C                      COEFFICIENT IS NOW LIMITED TO ABS(FCF1(4)).
C                      EACH COMPONENT HAS A FIELD WIDTH OF 12.
C
C     CARD 13.2
C     ---------
C
C ****************************************************************
C*    THIS CARD IS REQUIRED ONLY IF KTC.NE.0 ON CARD 6.          *
C ****************************************************************
C
C       IMOD    11-16  FILM THICKNESS COMPUTATION CODE --
C                         GRUBIN-S FORMULA              = 0
C                         DOWSON-HIGGINSON FORMULA      = 1
C                         HAMROCK-DOWSON FORMULA        = 2
C         HC    17-28  MINIMUM FILM FOR TRACTION MODEL BREAKDOWN (L).
C        STR    29-40  STARVATION PARAMETER - DISTANCE OF THE FILM
C                      BOUNDARY FROM THE EDGE OF THE CONTACT ZONE
C                      DIVIDED BY THE CONTACT HALF WIDTH (SEMI-MINOR
C                      AXIS IN CASE OF ELLIPTIC CONTACT).
C        TEM    41-64  VECTOR OF LENGTH 2 CONTAINING OUTER AND INNER
C                      RACE TEMPERATURES (TM).
C                      EACH COMPONENT HAS A FIELD WIDTH OF 12.
C
C     CARD 13.3
C     ---------
C
C ****************************************************************
C *   THIS CARD IS REQUIRED ONLY IF KTC=4 ON CARD 6.             *
C ****************************************************************
C
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C      XMU    11-22   VISCOCITY PARAMETER (F*T/L**2).                     RDREB515
C     GAMMA   23-34   PR-VIS PARAMETER (L**2/F).                          RDREB516
C      BETA   35-46   TEMP-VIS PARAMETER (1/TM).                          RDREB517
C       VR    47-58   CHARACTERISTIC VELOCITY (L/T) IN THE ROLLING        RDREB518
C                     SPEED EFFECT EXPRESSION.                            RDREB519
C       VN    59-70   EXPONENT IN THE ROLLING SPEED EFFECT EXPRESSION.    RDREB520
C                                                                         RDREB521
C    CARD 13.4                                                            RDREB522
C    ---------                                                            RDREB523
C                                                                         RDREB524
C ********************************************************************** RDREB525
C *  THIS CARD IS REQUIRED ONLY IF KTC=1 OR 4 ON CARD 6.                * RDREB526
C ********************************************************************** RDREB527
C                                                                         RDREB528
C       TO    11-22   INLET TEMPERATURE (TM).                             RDREB529
C     XMU0    23-34   INLET VISCOCITY (F*T/L**2).                         RDREB530
C      GAM    35-46   PR-VIS COEFFICIENT (L**2/F).                        RDREB531
C      BET    47-58   TEMP-VIS COEFFICIENT (TM).                          RDREB532
C       FK    59-70   THERMAL CONDUCTIVITY (F/T/TM).                      RDREB533
C                                                                         RDREB534
C    CARD 14                                                              RDREB535
C    -------                                                              RDREB536
C     ICHR    11-16   SWITCH FOR CHURNING EFFECTS --                      RDREB537
C                        CHURNING NEGLECTED           = 0                 RDREB538
C                        CHURNING CONSIDERED          = 1                 RDREB539
C                        EFFECTIVE VISCOCITY IS                           RDREB540
C                        DERIVED FROM LUB MODEL                           RDREB541
C                        (KTC.GT.0 ON CARD 6)         = 2                 RDREB542
C      CRO    17-28   EFFECTIVE LUB DENSITY (MA/L**3).                    RDREB543
C     VISC    29-40   EFFECTIVE VISCOCITY (F*T/L**2).  IF ICHR=2 ON       RDREB544
C                     THIS CARD THEN EFF VIS = VISC*(LUB MODEL VIS).      RDREB545
C    CARD 15.1                                                            RDREB546
C    ---------                                                            RDREB547
C                                                                         RDREB548
C ********************************************************************** RDREB549
C *  THIS CARD IS REQUIRED ONLY IF A CAGE IS PRESENT.                   * RDREB550
C *  KBT.GT.0 ON CARD 3.                                                * RDREB551
C ********************************************************************** RDREB552
C                                                                         RDREB553
C     FCF2    11-58   VECTOR OF LENTTH 4 CONTAINING THE TRACTION PARA-    RDREB554
C                     METERS FOR RE/CAGE INTERACTION.  THE PARAMETERS     RDREB555
C                     HAVE SAME DEFINITIONS AS FOR FCF1 ON CARD 13.1.     RDREB556
C                     EACH COMPONENT HAS A FIELD WIDTH OF 12.             RDREB557
C     HCBP    59-70   CRITICAL FILM (L) FOR RE/CAGE CONTACT.              RDREB558
C                                                                         RDREB559
C    CARD 15.2                                                            RDREB560
C    ---------                                                            RDREB561
C                                                                         RDREB562
C ********************************************************************** RDREB563
C *  THIS CARD IS REQUIRED ONLY IF A CAGE IS GUIDED ON THE RACE.        * RDREB564
C *  KBT.GT.0 ON CARD 3 AND IRGS(1)+IRGS(2).GT.0 ON CARD 9.1.          * RDREB565
C ********************************************************************** RDREB566
C                                                                         RDREB567
C     FCF3    11-58   VECTOR OF LENGTH 4 CONTAINING THE TRACTION PARA-    RDREB568
C                     METERS OF RACE/CAGE INTERACTION.  THE PARAMETERS    RDREB569
C                     HAVE SAME DEFINITIONS AS FOR FCF1 ON CARD 13.1.     RDREB570
C                     EACH COMPONENT HAS A FIELD WIDTH OF 12.             RDREB571
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C      HCCR   59-70   CRITICAL FILM (L) FOR RACE/CAGE CONTACT.
C
C    CARD 16
C    -------
C
C ****************************************************************
C *  THIS CARD IS REQUIRED ONLY IF A CAGE IS PRESENT,              *
C *  KBT.GT.0 ON CARD 3.                                          *
C ****************************************************************
C
C      CFOR   11-46   VECTOR OF LENGTH 3 CONTAINING THE X, Y AND Z
C                     COMPONENTS (INERTIAL FRAME) OF ANY EXTERNAL
C                     FORCE (F) VECTOR ACTING ON THE CAGE, SUCH AS
C                     LUBRICANT JET FORCE.
C                     EACH COMPONENT HAS A FIELD WIDTH OF 12.
C       RTC   47-58   RADIAL COORDINATE (L) RELATIVE TO THE BEARING
C                     CENTER AT WHICH THE ABOVE FORCE IS APPLIED.
C       THC   59-70   ORBITAL COORDINATE (DEG) DEFINING THE POSITION
C                     VECTOR LOCATING THE POINT OF APPLICATION OF
C                     CFOR IN THE INERTIAL FRAME.
C
C    CARD 17
C    -------
C
C ****************************************************************
C *  THIS CARD IS REQUIRED ONLY IF KTC=0 ON CARD 6, KBT.GT.0 ON   *
C *  CARD 3 AND IHBC+IHRC.GT.0 ON CARD 9.1.                       *
C ****************************************************************
C
C      VISP   11-22   EFF LUB VIS (F*T/L**2) FOR RE/CAGE INTERACTION.
C                     A ZERO MAY BE SPECIFIED IF IHBC=0 ON CARD 9.1.
C      VISR   23-34   EFF LUB VIS (F*T/L**2) FOR RACE/CAGE INTERACTION.
C                     A ZERO MAY BE SPECIFIED IF IHRC=0 ON CARD 9.1.
C
C    CARD 18
C    -------
C
C ****************************************************************
C *  THIS CARD IS REQUIRED ONLY IF MODE.GE.0 ON CARD 1.           *
C ****************************************************************
C
C    THE FOLLOWING CARD SPECIFIES RACE CONSTRAINTS FOR DYNAMIC
C    SOLUTIONS.  IRF AND IRM ARE TWO DIMENSIONAL ARRAYS (2,3).  THE
C    FIRST INDEX DENOTES THE RACE (OUTER=1, INNER=2) AND THE SECOND
C    INDEX DENOTES THE THREE COMPONENTS OF THE INERTIAL TRIAD --
C
C       IRF   11-28   LINEAR CONSTRAINT VECTOR OF LENGTH 6.  THE
C                     COMPONENTS ARE DEFINED AS --
C                        SPECIFIED FORCE              = 0
C                        SPECIFIED DISPLACEMENT       = 1
C                     EACH COMPONENT HAS A FIELD WIDTH OF 3.
C       IRM   29-46   ANGULAR CONSTRAINT VECTOR OF LENGTH 6.  THE
C                     COMPONENTS ARE DEFINED AS --
C                        SPECIFIED MOMENT             = 0
C                        SPECIFIED ANG DISPLACEMENT = 1
C                     EACH COMPONENT HAS A FIELD WIDTH OF 3.
C
```

```
C    CARD 19
C    -------
C
C *********************************************************************
C *  THIS CARD IS REQUIRED ONLY IF MODE.GE.0 ON CARD 1.              *
C *********************************************************************
C
C          G    11-46    VECTOR OF LENGTH 3 CONTAINING THE COMPONENTS OF
C                        THE ACCELERATION (L/T**2) DUE TO GRAVITY VECTOR
C                        IN THE INERTIAL FRAME.  EACH COMPONENT HAS A
C                        FIELD WIDTH OF 12.
C
C
C *********************************************************************
C *  THE FOLLOWING SERIES 20 CARDS ARE REQUIRED ONLY IF MODE=2 ON     *
C *  CARD 1.                                                          *
C *********************************************************************
C
C    CARD 20.1
C    ---------
C         TI    11-25    INITIAL DIMENSIONLESS TIME.
C
C    CARDS 20.2 TO CARD 20.N
C    -----------------------
C          X    11-70    THE DIMENSIONLESS SOLUTION VECTOR OF LENGTH
C                        (30+12*NB+6*KBT/ABS(KBT)) WHICH WILL BE USED AS
C                        INITIAL CONDITIONS.  EACH CARD WILL HAVE FOUR
C                        COMPONENTS, EACH ONE HAVING A FIELD WIDTH OF 15.
C
C    CARDS 21 TO ......
C    ------------------
C
C    ANY INPUT DATA REQUIRED BY THE USER ACCESSIBLE ROUTINES -ALOAD-,
C    -PSPRG- AND -RSPRG- SHOULD BE SUPPLIED IN ORDER.  (SEE DISCUSSION
C    OF SPECIAL ROUTINES BELOW).
C
C
C    OUTPUT DATA
C    ===========
C
C    OUTPUT SOLUTIONS MAY BE EITHER PRINTED, PLOTTED OR BOTH.  AS DIS-
C    CUSSED UNDER THE INPUT DESCRIPTIONS SEVERAL DIFFERENT OPTIONS FOR
C    MONITORING THE OUTPUT ARE AVAILABLE.  ALL THE OUTPUT IS VERY WELL
C    DOCUMENTED IN ORDER TO MAKE IT SELF-EXPLANATORY.  ALL DIMENSIONAL
C    VARIABLES ARE PRINTED WITH PROPER UNITS.
C
C
C    FILE SYSTEM
C    ===========
C
C    THE PROGRAM HAS FULL RESTART CAPABILITIES AND IT CAN MONITOR THE
C    MOTION OF A TOTAL OF SIX BEARING ELEMENTS INCLUDING THE CAGE AND
C    THE RACEWAYS.  THE ELEMENT SELECTION IS DESCRIBED UNDER THE INPUT
C    DESCRIPTION AND SOME DETAILS OF THE FILING SYSTEM ARE DISCUSSED
C    BELOW.
C
C    FILE ESOL
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C     ---------
C     THIS IS THE MASTER DATA FILE WHICH CONTAINS ALL THE BEARING
C     INFORMATION AND SOLUTION VECTORS AT SELECTED TIME STEPS.  THIS
C     FILE MUST BE SAVED IF THE EXECUTION HAS TO BE RESTARTED LATER.
C
C     FILE FSOL
C     ---------
C     AT THE END OF A SUCCESSFUL RUN THE LAST SOLUTION VECTOR IS
C     WRITTEN IN THIS FILE IN A FORMAT SUCH THAT THE DATA COULD BE
C     PUNCHED ON CARDS AND DIRECTLY USED UNDER THE ARBITRARY INITIAL
C     CONDITIONS OPTION.  THIS FILE IS COMPLETELY REWRITTEN AT THE
C     END OF EACH EXECUTION OF THE PROGRAM.
C
C     FILES GSOL1 TO GSOL6
C     --------------------
C     THESE SIX FILES STORE ALL SOLUTIONS FOR THE SIX PRESELECTED BEAR-
C     ING ELEMENTS.  IF THE NUMBER OF ELEMENTS SELECTED IS LESS THAN SIX
C     THEN THE FIRST FILE NUMBERS ARE FILLED.  IN OTHER WORDS IF THERE
C     ARE ONLY THREE ELEMENTS MONITORED THEN THE SOLUTIONS ARE CONTAIN-
C     ED IN FILES GSOL1 TO GSOL3 AND THE FILES GSOL4 TO GSOL6 REMAIN
C     EMPTY.  WHEN A COMPLETE CASE CONSISTS OF SEVERAL EXECUTIONS OF
C     THE PROGRAM, THEN IT WILL BE NECESSARY TO ATTACH ALL THE FILES
C     USED WITH THE PROPER WRITE PERMISSION SO THAT SOLUTIONS FOR THE
C     ENTIRE TIME RANGE MAY BE STORED FOR PLOTTING PURPOSES.
C
C     FILE HSOL
C     ---------
C     THE OVER ALL BEARING PARAMETERS, WHICH CONSISTS OF LOAD*SLIP,
C     TRACTION*SLIP, AND TORQUE VARIATIONS, ARE STORED IN THIS FILE AS
C     A FUNCTION OF TIME.
C
C     ALL DATA FILES CONTAIN APPROPRIATE IDENTIFICATION CODES, WHICH ARE
C     CHECKED WHENEVER THE FILES ARE ATTACHED.  THE FILES PRODUCED WITH
C     TWO DIFFERENT RUNS, THEREFORE, MAY NOT BE INTERCHANGED.
C
C
C     SPECIAL ROUTINES
C     ================
C
C     THE FOLLOWING SPECIAL ROUTINES ARE AVAILABLE TO THE USER IN ORDER
C     TO CLOSELY SIMULATE THE APPLICATION UNDER QUESTION --
C
C       1.  APLOAD    - APPLIED LOAD
C       2.  PSPRG     - ARBITRARY SPRING AT BALL/CAGE CONTACT
C       3.  RSPRG     - ARBITRARY SPRING AT RACE/CAGE CONTACT
C
C     SEE DOCUMENTATION SUPPLIED WITH THE ABOVE ROUTINES FOR NOTES ON
C     PROGRAMMING THE FEATURES OF A GIVEN APPLICATION IN THE ABOVE
C     SUBROUTINES.
C
C
C     EXTERNALS CALLED
C     ================
C
C     THE FOLLOWING LIST OF SUBROUTINES INCLUDES THE ROUTINES CALLED
C     DIRECTLY BY -RDREB- AND ALSO THOSE CALLED BY THE ROUTINES
C     INITIALLY CALLED BY -RDREB-  ---
```

```
C                                                                        RDREB743
C         1.  ABM       - PREDICTOR-CORRECTOR ALGORITHM                  RDREB744
C         2.  ACCN      - ANGULAR ACCELERATION TRANSFORMATION            RDREB745
C         3.  ARTAN     - MODIFIED ARCTAN                                RDREB746
C         4.  BCCON     - BALL/CAGE INTERACTION                          RDREB747
C         5.  BGEOM     - BEARING GEOMETRY                               RDREB748
C         6.  BLOAD     - BALL/RACE LOAD                                 RDREB749
C         7.  BTRAC     - BALL/RACE TRACTION                             RDREB750
C         8.  CELIL     - ELLIPTIC INTEGRALS                             RDREB751
C         9.  CHURN     - CHURNING MOMENTS                               RDREB752
C        10.  CRCON     - CAGE/RACE INTERACTION                          RDREB753
C        11.  CURF      - CURVATURE FUNCTION FOR ELASTIC CONTACT         RDREB754
C        12.  DERIV     - COMPUTATION OF DERIVATIVES                     RDREB755
C        13.  ELCON     - ELASTICITY SOLUTION                            RDREB756
C        14.  FCOEF     - TRACTION COEFFICIENT                           RDREB757
C        15.  FCT       - CURVATURE FUNCTION FOR ELASTIC CONTACT         RDREB758
C        16.  FCTT      - CAGE/RACE CONTACT COORDINATE FUNCTION          RDREB759
C        17.  FLB       - LONG BEARING FUNCTION                          RDREB760
C        18.  FLIFE     - FATIGUE LIFE COMPUTATION                       RDREB761
C        19.  FPHI      - RACE AXIMUTH                                   RDREB762
C        20.  FRIC      - FRICTION FORCE FUNCTION                        RDREB763
C        21.  FSB       - SHORT BEARING FUNCTION                         RDREB764
C        22.  FTRS      - TRACTION PARAMETER FUNCTION                    RDREB765
C        23.  FUNC      - INTEGRAND FOR TRACTION INTEGRAL                RDREB766
C        25.  HYDRO     - HYDRODYNAMICS FOR SHORT AND LONG BEARING MODELS RDREB767
C        26.  IANG      - INITIAL ANGULAR VELOCITIES                     RDREB768
C        27.  PCOEFF    - POLYNOMIAL COEFFICIENTS FOR PREDICTOR-CORRECTOR RDREB769
C        28.  PHITS     - THERMAL AND SIDE LEAKAGE FACTORS               RDREB770
C        29.  PLOAD     - PALMGREN FORMULA                               RDREB771
C        30.  POUT      - PRINT OUTPUT                                   RDREB772
C        31.  PRVIT     - PRESSURE-VISCOCITY-TEMPERATURE RELATION        RDREB773
C        32.  QG10      - TENTH ORDER GAUSSIAN QUADRATURE INTEGRATION    RDREB774
C        33.  QSTAT     - QUASI-STATIC EQUILIBRIUM                       RDREB775
C        34.  RKM4      - EXPLICIT INTEGRATING ALGORITHMS                RDREB776
C        35.  ROLLF     - ROLLING FUNCTION                               RDREB777
C        36.  RTMI      - ROOT EVALUATION FOR A GIVEN FUNCTION           RDREB778
C        37.  SLIP      - SLIP VELOCITY BETWEEN TWO INTERACTING BODIES   RDREB779
C        38.  STEQM     - STATIC EQUILIBRIUM                             RDREB780
C        39.  TRAC      - TRACTION ON INCREMENTAL AREA                   RDREB781
C        40.  TRCF      - TRACTION DATA FOR MINERAL OILS                 RDREB782
C        41.  TRFM      - TRANSFORMATION MATRIX                          RDREB783
C        42.  TRMOD     - EHD TRACTION MODELS                            RDREB784
C        43.  TRSLP     - COEFFICIENTS FOR SIMPLIFIED TRACTION RELATION  RDREB785
C        44.  VELC      - ANGULAR VELOCITY TRANSFORMATION                RDREB786
C        45.  VPDT      - VECTOR PRODUCT                                 RDREB787
C        46.  ZINT      - LINEAR INTERPOLATION                           RDREB788
C        47.  ALOAD     - APPLIED LOAD                                   RDREB789
C        48.  PSPRG     - ARBITRARY SPRING AT BALL/CAGE CONTACT          RDREB790
C        49.  RSPRG     - ARBITRARY SPRING AT RACE/CAGE CONTACT          RDREB791
C                                                                        RDREB792
C                                                                        RDREB793
C      NOTES                                                             RDREB794
C      =====                                                             RDREB795
C                                                                        RDREB796
C      (1)    A MAXIMUM OF 40 ROLLING ELEMENTS ARE ALLOWED.  IF THE ACTUAL RDREB797
C             NUMBER EXCEEDS 40, THEN ALL DIMENSION DECLARATIONS, INCLUD- RDREB798
C             ING THOSE IN THE COMMON BLOCKS WILL HAVE TO BE CHANGED    RDREB799
C             ACCORDINGLY.                                               RDREB800
C                                                                        RDREB801
```

```
C     (2)    THE PROGRAM WITH ALL THE ROUTINES USES A CENTRAL MEMORY OF
C            ABOUT 158000B WORDS.
C
C     (3)    THIS PROGRAM HAS BEEN DEVELOPED ON THE CDC SYSTEM WHICH
C            EMPLOYS 60 BIT WORDS.  ON OTHER SYSTEMS WHERE SHORTER WORDS
C            ARE USED IT MAY BE NECESSARY TO USE DOUBLE PRECISION.
C
C     (4)    FOR THE PURPOSE OF ROOT FINDING AND TEST AGAINST ZEROS THE
C            PROGRAM USES TOLERANCES WHICH RESIDE IN THE ARRAY TOLR IN
C            THE COMMON BLOCK TCOMA.  THE ARRAY HAS THREE COMPONENTS.
C            THE FIRST AND SECOND COMPONENTS ARE TOLERANCES FOR ABSOLUTE
C            AND RELATIVE ZEROS RESPLY.  THE THIRD COMPONENT IS A
C            TOLERANCE ON ACCELERATION COMPONENTS.  IF ANY EXT FORCE
C            HAS AN ABSOLUTE VALUE LESS THAN THIS TOLERANCE THEN IT IS
C            SET EQUAL TO ZERO.  PROPER VALUES TO THE ARRAY TOLR ARE
C            ASSIGNED IN THE BEGINNING OF THE MAIN PROGRAM -DREB-.
C
C     (5)    TIME STEP SIZE OPTIMIZATION IS PROVIDED BY NOPT VECTOR AND
C            THE VALUE OF TOL IN -RDREB-.  TOL SHOULD BE 1 TO 2 ORDERS
C            OF MAGNITUDE GREATER THAN THE TOLERANCE ON RELATIVE ZEROS
C            DISCUSSED ABOVE.  IF THE TRUNCATION AT ANY STEP EXCEEDS TOL
C            THEN THE STEP SIZE IS SUCCESSIVELY HALVED UNTIL EITHER THE
C            TOLERANCE CRETERION IS MET OR THE MIN STEP SIZE, SPECIFIED
C            ON CARD 2, IS REACHED.  IF THE TOLERANCE CRETERION IS NOT
C            MET FOR NOPT(1) CONSECUTIVE STEPS THEN THE EXECUTION IS TER-
C            MINATED.  A TYPICAL VALUE FOR NOPT(1) MAY BE ABOUT 3 TO 6.
C            IF THE TOLERANCE CRITERION IS MET FOR NOPT(2) SUCCESSIVE
C            STEPS THEN AN ATTEMPT IS MADE TO SUCCESSIVELY DOUBLE THE
C            STEP SIZE UNTIL EITHER THE MAX STEP SIZE OR AN UNACCEPTABLE
C            TRUNCATION ERROR IS REACHED.  THE OPTIMUM STEP IS THEN
C            SELECTED FOR THE NEXT NOPT(2) STEPS.  A TYPICAL VALUE FOR
C            NOPT(2) MAY BE 10.  IF NO OPTIMIZATION IS REQUIRED THEN
C            NOPT(2) MAY BE SET TO A VALUE GREATER THAN THE MAX NUMBER
C            OF STEPS IN THE RUN, WHICH IS SPECIFIED AS NOPT(3).  THE
C            MAXIMUM NUMBER OF STEPS SHOULD BE SELECTED ON THE BASIS OF
C            THE EXPECTED COMPUTING TIME PER STEP AND THE PERMISSIBLE
C            TIME LIMIT ON THE JOB.  ADEQUATE ALLOWANCE MUST BE MADE IN
C            ORDER TO PREVENT -TIME LIMIT- ERROR, WHICH ON SOME SYSTEMS
C            MAY RESULT IN IMPROPER CLOSURE OF DATA FILES AND HENCE A
C            SUBSTANTIAL PORTION OF THE OUTPUT MAY BE LOST.
C
C     (6)    FOR MOST BEARINGS A TIME STEP SIZE OF ABOUT 10 MICROSECS
C            WILL BE ADEQUATE FOR CONVERGENCE TO WITHIN REASONABLE
C            TOLERANCE LIMITS.  A TRIAL RUN WILL PRINT THE RELEVANT TIME
C            SCALE WHICH CAN BE USED TO ESTIMATE THE DIMENSIONLESS STEP
C            SIZE.  THE MIN AND MAX STEP SIZES, SPECIFIED AS DMIN AND
C            DMAX RESPLY ON CARD 2, MAY BE ESTIMATED BY THE TRANSIENT
C            DETAILS REQUIRED IN THE SIMULATIONS.  THE INITIAL STEP
C            SIZE, DINT, MUST BE BETWEEN DMIN AND DMAX.  IF DINT=DMIN=
C            DMAX THEN NO STEP OPTIMIZATION WILL BE PERFORMED AND UNDER
C            SUCH A CONDITION NOPT(2) ON CARD 1 SHOULD BE SET EQUAL TO
C            A VALUE GREATER THAN THE MAX NUMBER OF STEPS IN THE RUN.
C
C
C     PRADEEP K. GUPTA
C     VERSION RAPIDREB.0                JULY 1981
C
C-----------------------------------------------------------------------
C
```

```
C
C-----------------------------------------------------------------------
C
C                        PROGRAM RDREBP
C                        --------------
C
C                     RAPID SIMULATION OF THE
C                     =======================
C
C          DYNAMICS OF ROLLING ELEMENT BEARINGS (PLOT PROGRAM)
C          ======== == ======= ======= ======== ==============
C
C        -PLOT OUTPUT FOR THE REAL TIME PERFORMANCE SIMULATION-
C
C                                BY
C
C                          PRADEEP K. GUPTA
C                MECHANICAL TECHNOLOGY INCORPORATED
C                      968 ALBANY-SHAKER ROAD
C                          LATHAM, NEW YORK
C                             U. S. A.
C
C                           SPONSORED BY
C                 WRIGHT-PATTERSON AIR FORCE BASE
C                               OHIO
C                   CONTRACT NO. F33615-80-C-5152
C
C
C    ALL OUTPUT SOLUTIONS GENERATED BY THE DYNAMICS OF ROLLING ELEMENT
C    ALL OUTPUT SOLUTION GENERATED BY THE PROGRAM -RDREB- (RAPID SIMU-
C    LATION OF THE DYNAMICS OF ROLLING ELEMENT BEARINGS) ARE PLOTTED BY
C    THIS PROGRAM.  THE DATA FILE CONTAINING THE SOLUTIONS FOR A GIVEN
C    BEARING ELEMENT IS ATTACHED BEFORE EXECUTING -RDREBP-.  TO MINI-
C    MIZE THE CORE REQUIREMENT THIS MASTER DATA FILE IS SUBDIVIDED INTO
C    SEVERAL LOCAL FILES SUCH THAT EACH LOCAL FILE CONTAINS ALL THE
C    DATA FOR A PARTICULAR PLOT.  THE LOCAL FILES ARE THEN PLOTTED IN
C    SEQUENCE.  AS DISCUSSED BELOW THE PROGRAM HAS FULL CAPABILITIES OF
C    AUTOMATIC SCALING TO ANY DESIRED DEGREE.
C
C    INPUT DESCRIPTION
C    =================
C
C    ALL DATA IS SUPPLIED IN COLUMNS 11 TO 70.  THE FIRST TEN AND THE
C    LAST TEN COLUMNS ARE AVAILABLE FOR ANY IDENTIFICATION PURPOSE.
C    THE FOLLOWING DESCRIPTION SUMMARIZES THE INPUT VARIABLE NAMES,
C    THEIR FIELDS ON THE DATA CARDS AND THE RELEVANT DESCRIPTION, IN
C    ORDER.  VARIABLES BEGINNING WITH THE LETTERS I, J, K, L, M OR N
C    ARE INTEGERS AND ALL OTHERS ARE REAL NUMBERS.  THE FORMAT OF ALL
C    REAL NUMBERS IS E12.5 AND THAT FOR THE INTEGERS IS DETERMINED BY
C    THE SPECIFIED FIELD LENGTH AND THE NUMBER IS RIGHT JUSTIFIED IN
C    ITS FIELD.
C
C    CARD 1
```

```
C     ------                                                            
C     ITP        11-12  SWITCH FOR PLOTTING TITLE PAGE --
C                       ITP        =0  PLOT TITLE PAGE.
C                                  =1  SUPPRESS TITLE PAGE PLOTTING.
C     JTP        13-16  NUMBER OF POINTS TO BE SKIPPED IN THE PLOT
C                       JTP=0 WILL PLOT ALL POINTS.  THE TOTAL NUMBER OF
C                       POINTS TO BE PLOTTED MUST BE .LE. 3000
C     JRI        17-23  RECORD NUMBER FROM WHICH PLOT SHOULD START.
C     JRF        24-30  RECORD NUMBER AT WHICH PLOT SHOULD END.
C     IPLT       31-70  A VECTOR WHOSE LENGTH IS EQUAL TO THE NUMBER OF
C                       PLOTS FOR THE BEARING ELEMENT UNDER CONSIDERAT-
C                       ION.  THERE COULD BE A MAXIMUM OF TEN PLOTS.
C                       THE VALUE ASSIGNED TO EACH COMPONENT HAS THE FOL-
C                       LOWING SIGNIFICANCE --
C                       IPLT(I)    =0  ITH PLOT IS SUPPRESSED.
C                                  =1  AUTOMATIC SCALING IS USED ON THE ITH
C                                      PLOT.
C                                  =2  SCALE INFORMATION IS SUPPLIED IN THE
C                                      FOLLOWING CARDS.
C                       EACH COMPONENT HAS A FIELD WIDTH OF 4.
C
C     IF NONE OF THE COMPONENTS OF IPLT ARE SET EQUAL TO 2 THEN NO AD-
C     DITIONAL DATA CARDS ARE NECESSARY.  IF THE VALUE 2 IS ASSIGNED TO
C     ANY COMPONENT THEN CARDS 2 TO 5 MUST BE SUPPLIED FOR EACH COMPON-
C     ENT OF IPLT (=2) IN ORDER.
C
C     CARD 2
C     ------
C     ISCX(I)    11-15  TIME AXIS SCALE CODE WITH FOLLOWING SIGNIFI-
C                       CANCE --
C                       ISCX(I)    =0  AUTOMATIC SCALE FOR TIME AXIS.
C                                  =1  SCALE TIME AXIS ACCORDING TO SPEC-
C                                      IFIED PARAMETERS.
C     XIX(I)     16-25  MINIMUM VALUE OF TIME.
C     XFX(I)     26-35  MAXIMUM VALUE OF TIME.
C     NXX(I)     36-40  SCALE FACTOR (IN TERMS OF 10**NXX) TO BE USED FOR
C                       THE TIME AXIS.
C
C     CARD 3            (SEE NOTE (5) FOR CAGE MOTION PLOTS)
C     ------
C     ISCY(1,I)  11-15  SCALE CODE FOR THE Y AXIS NO. 1 WITH THE FOLLOW-
C                       ING SIGNIFICANCE --
C                       ISCY(1,I)=0  AUTOMATIC SCALE FOR Y AXIS NO. 1.
C                                  =1  SCALE Y-AXIS NO.1 AS SPECIFIED AND
C                                      SKIP AXIS IF DATA OUT OF SCALE.
C                                  =2  SCALE Y-AXIS NO.1 AS SPECIFIED AND
C                                      PLOT EVEN IF DATA OUT OF SCALE.
C                                  =3  SCALE Y-AXIS NO.1 AS SPECIFIED AND
C                                      TRUNCATE DATA IF OUT OF SCALE.
C     YIY(1,I)   16-25  MINIMUM VALUE OF THE VARIABLE TO BE PLOTTED ON Y
C                       AXIS NO. 1.
C     YFY(1,I)   26-35  MAXIMUM VALUE OF THE VARIABLE TO BE PLOTTED ON Y
C                       AXIS NO. 1.
C     NYY(1,I)   36-40  SCALE FACTOR (IN TERMS OF 10**NXX) TO BE USED FOR
C                       THE X AXIS.
C
C     CARDS 4 AND 5
```

```
C     -------------
C     SIMILAR TO CARD 3 FOR Y AXIS NUMBERS 2 AND 3.
C     THE Y AXES ARE NUMBERED FROM THE BOTTOM TO THE TOP OF THE PLOT.
C
C
C     DATA FILE MANAGEMENT
C     ====================
C
C     THE FOLLOWING DATA FILES ARE USED DURING EXECUTION OF DREBP --
C
C     SOL   -  THE MASTER DATA FILE CONTAINING ALL THE SOLUTIONS FOR THE
C              GIVEN BEARING ELEMENT.
C     SOL1
C      TO   -  LOCAL FILES CONTAINING RESPECTIVE INFORMATION FOR PLOT
C     SOL10    NO. 1 TO 10
C
C     A MAXIMUM OF TEN PLOTS IS PRODUCED BY -RDREBP-.  THE ACTUAL NUMBER
C     OF PLOTS MAY, HOWEVER, DEPEND ON THE PARTICULAR MASTER
C     DATA FILE SOL WHICH IDENTIFIES THE BEARING ELEMENT AND HENCE THE
C     REQIRED PLOTS.
C
C     OUTPUT DESCRIPTION
C     ==================
C
C     THE PRIMARY OUTPUT OF DREBP CONSISTS OF THE VARIOUS PLOTS FOR THE
C     DIFFERENT BEARING PARAMETERS.  AS DISCUSSED ABOVE THE BEARING
C     PARAMETERS AND THE NUMBER OF PLOTS ARE DETERMINED FROM THE DATA
C     SUPPLIED BY THE MASTER DATA FILE SOL.  IN GENERAL FOR THE VARIOUS
C     DATA FILES GENERATED BY DREB, THE NUMBER OF PLOTS GENERATED BY
C     DREBP ARE INDICATED BELOW --
C
C     MOTION OF ROLLING ELEMENT                            10 PLOTS
C     MOTION OF CAGE                                        9 PLOTS
C     MOTION OF OUTER RACE                                  6 PLOTS
C     MOTION OF INNER RACE                                  6 PLOTS
C     OVERALL BEARING PARAMETERS                            1 PLOT
C
C     EACH SET OF PLOTS HAS A TITLE PAGE WHICH CONTAINS THE BEARING IN-
C     FORMATION AND ALL THE SCALE FACTORS FOR THE VARIOUS PARAMETERS.
C
C     ALONG WITH ALL THE PLOTS DREBP ALSO PRODUCES A BRIEF PRINT OUTPUT.
C     A LISTING OF ALL DATA CARDS AS SUPPLIED BY THE USER IS PRINTED,
C     FOR DIAGNOSTIC PURPOSES, AS THE CARDS ARE READ IN.  THE ESSENTIAL
C     PRINT OUTPUT CONSISTS OF THE BEARING IDENTIFICATION, FUNDAMENTAL
C     SCALE FACTORS AND A DETAILED TABLE CONTAINING ALL THE SCALE IN-
C     FORMATION FOR THE VARIOUS PLOTS.  THE EXECUTION OF A PRINT STATE-
C     MENT CORRESPONDING TO A GIVEN PLOT NUMBER ALSO INDICATES A SUC-
C     CESSFUL PLOT.
C
C     EXTERNALS CALLED
C     ================
C
C     OTHER THAN THE CONVENTIONAL CALCOMP SOFTWARE, THE FOLLOWING EX-
C     TERNAL ROUTINES MUST BE SUPPLIED.  THE LIST INCLUDES THE ROUTINES
C     CALLED DIRECTLY BY DREBP OR THE ONES CALLED BY ANY ROUTINE INIT-
C     IALLY CALLED BY DREBP.
C
```

```
C       1.  AXC       - COMPUTE SCALE FACTORS FOR GIVEN AXIS.
C       2.  FNUMBR    - PLOT REAL NUMBER IN CONVENTIONAL E FORMAT.
C       3.  MINMAX    - FIND MINIMUM AND MAXIMUM VALUES FOR A GIVEN AXIS.
C       4.  MTI       - PLOT MTI INSIGNIA.
C       5.  SFAC      - PLOT SCALE FACTOR ON A GIVEN AXIS.
C       6.  SIGF      - EXTRACT SIGNIFICANT FIGURES OF A REAL NUMBER.
C       7.  TITLE     - SET-UP ALL PLOT TITLES.
C
C     NOTES
C     =====
C
C     (1)   A MAXIMUM OF 3000 POINTS MAY BE PLOTTED IN A GIVEN CURVE.
C           IF THE NUMBER OF POINTS .GT. 3000 THEN EITHER THE DIMENSION
C           STATEMENTS IN THE UNLABELLED COMMON (APPEARING IN THE MAIN
C           PROGRAM AND THE SUBROUTINE -MINMAX-) MUST BE CHANGED OR THE
C           START, FINISH AND NUMBER OF POINTS TO BE SKIPPED SHOULD BE
C           ADJUSTED (SEE CARD 1) SO THAT NO MORE THAN 3000 POINTS ARE
C           PLOTTED.
C
C     (2)   APPROPRIATE VALUES FOR -MACH-, -TAB- AND -SMAR- MUST BE
C           SET IN THE DATA STATEMENT FOR PROPER PROGRAM OPERATION --
C              MACH = 1, TEN CHARACTERS PER WORD FOR THE CDC SYSTEM.
C              MACH = 2, FOUR CHARACTERS PER WORD FOR THE IBM SYSTEM.
C              TAB  =     WIDTH (INCH) OF PAPER BETWEEN TEAR LINES IF
C                         ANY.  TAB MUST BE .GE.8.5.  ALL PLOTS ARE
C                         DESIGNED TO FIT ON A 8.5 INCH WIDE PAPER.
C              SMAR =     WIDTH OF MARGIN USED BY THE SYSTEM FOR USER
C                         IDENTIFICATION.  FOR NO SYSTEM MARGIN, SMAR=0.
C
C     (3)   THE FIRST FOUR COMPONENTS OF ARRAY -IFOR-, DEFINED IN A
C           DATA STATEMENT, ARE USED ONLY WITH MACH=1 FOR THE CDC
C           SYSTEM.  IF THE 10H FIELD IN THE DATA STATEMENT IS UNACCEPT-
C           ABLE ON THE IBM SYSTEM WITH MACH=2 THEN THE VALUES FOR THE
C           FIRST FOUR COMPONENTS MAY BE TRUNCATED BUT UNDER NO CIRCUM-
C           STANCE THE DIMENSIONS OF -IFOR- MAY BE ALTERED.
C
C     (4)   THE FORM OF THE CALCOMP INITIALIZATIONN ROUTINE -PLOTS- MUST
C           BE CHECKED FOR THE GIVEN INSTALLATION AND THE STATEMENT 72
C           IN -RDREBP- MUST BE ADJUSTED ACCORDINGLY.
C
C     (5)   PLOT NO. 9 FOR THE CAGE MOTION IS A CAGE MASS CENTER ORBIT
C           PLOT AND IT CONTAINS ONLY TWO AXES CORRESPONDING TO THE Y
C           AND Z POSITION OF THE CAGE MASS CENTER.  IF ANY SCALING
C           INFORMATION IS SUPPLIED FOR THIS PLOT THEN THE DATA FOR Y
C           POSITION SHOULD BE SUPPLIED ON CARD 2 AND THE DATA FOR Z
C           POSITION SHOULD BE SUPPLIED ON CARD 3, WHICH IS NORMALLY
C           USED FOR Y AXIS NO. 1.  ALSO IN THIS CASE ISCY(1,I) MAY BE
C           EQUAL TO ONLY A 0 OR 1.  THE OTHER TWO OPTIONS ARE NOT
C           AVAILABLE FOR THIS PLOT.
C
C     (6)   -RDREBP- REQUIRES A CENTRAL MEMORY OF ABOUT 164000B WORDS
C           ON A CDC SYSTEM.
C
C
C     PRADEEP K. GUPTA
C     VERSION   RAPIDREBP1                          SEPTEMBER 1981
C
C-----------------------------------------------------------------------
C
```

APPENDIX B

TYPICAL COMPUTER OUTPUT FOR THE
100 mm AND 150 mm DMA BEARINGS

```
RRRRRRRRRRRRRRR    DDDDDDDDDDDDDDD    RRRRRRRRRRRRRRR    EEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
RRRRRRRRRRRRRRRR   DDDDDDDDDDDDDDDD   RRRRRRRRRRRRRRRR   EEEEEEEEEEEEEEE    BBBBBBBBBBBBBBBB
  RR         RR      DD        DD       RR         RR    EE                   BB         BB
  RR         RR      DD        DD       RR         RR    EE                   BB         BB
  RRRRRRRRRRRR       DD        DD       RRRRRRRRRRRR     EEEEEEEEEE           BBBBBBBBBBBB
  RRRRRRRRRRR        DD        DD       RRRRRRRRRRR      EEEEEEEEEE           BBBBBBBBBBBB
  RR      RR         DD        DD       RR      RR       EE                   BB         BB
  RR       RR        DD        DD       RR       RR      EE                   BB         BB
  RR        RR     DDDDDDDDDDDDDDDD     RR        RR     EEEEEEEEEEEEEEEE   BBBBBBBBBBBBBBBB
  RR         RR    DDDDDDDDDDDDDDD      RR         RR    EEEEEEEEEEEEEEEE   BBBBBBBBBBBBBBB
```

R A P I D   S I M U L A T I O N   O F   T H E

D Y N A M I C S   O F   R O L L I N G   E L E M E N T   B E A R I N G S

-A REAL TIME PERFORMANCE SIMULATION-
(VERSION RAPIDREB.0)

PRADEEP K. GUPTA
MECHANICAL TECHNOLOGY INCORPORATED
968 ALBANY-SHAKER ROAD
LATHAM, NEW YORK
U. S. A.

BEARING TYPE -- BALL   SPEC CODE -- 100MM-DMA-BRG IRG-CAGE 60RPM 178N**

BEARING GEOMETRY

BORE	(M) =	1.00000E-01
OUTSIDE DIAMETER	(M) =	1.50000E-01
SHAFT INNER DIA	(M) =	8.00000E-02
BALL DIAMETER	(M) =	1.58750E-02
PITCH DIAMETER	(M) =	1.25000E-01
CONTACT ANGLE	(DEG) =	2.60000E+01
END PLAY	(M) =	5.56731E-04
CAGE OUTER DIA	(M) =	1.35000E-01
CAGE INNER DIA	(M) =	1.15000E-01
EFF CAGE WIDTH	(M) =	2.60000E-02

OUTER RACE SHRINK FIT	(M) =	1.00000E-05
INNER RACE SHRINK FIT	(M) =	1.00000E-05
HOUSING OUTER DIA	(M) =	2.00000E-01
NUMBER OF BALLS	=	19
OUTER RACE CUR FACTOR	=	5.20000E-01
INNER RACE CUR FACTOR	=	5.20000E-01
DIAMETRAL PLAY	(M) =	1.28532E-04
CAGE OUTER DIA CLS	(M) =	3.50000E-03
CAGE INNER DIA CLS	(M) =	4.00000E-04
DIA RF/CAGE CLEARANCE	(M) =	3.00000E-04

	GUIDANCE TYPE	GUIDING RACE RADIUS (M)	GUIDING CAGE RADIUS (M)	EFF LAND WIDTH (M)	CAGE HALF WIDTH (M)
GUIDING LAND I	2	5.73000E-02	5.75000E-02	5.00000E-03	1.30000E-02
GUIDING LAND II	2	5.73000E-02	5.75000E-02	5.00000E-03	1.30000E-02

```
LUBRICATION DETAILS
-------------------

 1. ROLLING ELEMENT/CAGE AND RACE/CAGE PARAMETERS ---

                                           INTERACTION  LUB VISCOCITY      MAX FILM
                                               CODE       (N*S/M**2)          (M)

    ROLLING ELEMENT/CAGE INTERACTION            0        0.            1.00000E-04
    RACE/CAGE INTERACTION                       0        0.

    RE/CAGE DRY CONTACT TRACTION PARAMETERS --

    CRIT FILM THICKNESS   (M)     =  2.50000E-07
    COEFFICIENT A                 =  1.00000E-02     COEFFICIENT B    1/ (M/S) =  0.
    COEFFICIENT C       1/ (M/S)  =  0.              COEFFICIENT D             = -1.00000E-02

    CAGE/RACE DRY CONTACT TRACTION PARAMETERS --

    CRIT FILM THICKNESS   (M)     =  2.50000E-07
    COEFFICIENT A                 =  1.00000E-02     COEFFICIENT B    1/ (M/S) =  0.
    COEFFICIENT C       1/ (M/S)  =  0.              COEFFICIENT D             = -1.00000E-02

 2. ROLLING ELEMENT/RACE PARAMETERS (TRACTION CODE = 0) ---

    COEFFICIENT A                 =  0.              COEFFICIENT B    1/ (M/S) =  4.10000E-02
    COEFFICIENT C       1/ (M/S)  =  0.              COEFFICIENT D             = -1.00000E-02
```

```
APPLIED LOADS AND SPEEDS
------------------------

 1. QUASI-STATIC SIMULATION ---

    AXIAL  LOAD                (N) =  1.78000E+02      OUTER RACE ANG VEL (RPM) =  0.
    RADIAL DISPLACEMENT        (M) =  0.               INNER RACE ANG VEL (RPM) =  6.00000E+01
    RELATIVE MISALIGNMENT (DEG) =  0.

 2. DYNAMIC SIMULATION ---
                      *************** LOADS    (N) ************        ******** MASS CEN ACC    (M/S**2) ********
                                   I              II             III                  I               II              III

    OUTER RACE                                                          0.              0.              0.
    INNER RACE                                                          0.              0.              0.

                      ************ MOMENTS    (N*M) ***********        ******** ANG ACCELERATION (RPM/S) *******
                                   I              II             III                  I               II              III

    OUTER RACE                                                          0.              0.              0.
    INNER RACE                                                          0.              0.              0.

                      ******** ANGULAR VELOCITIES (RPM)********
                                   I              II             III

    OUTER RACE        0.               0.              0.
    INNER RACE        6.00000E+01      0.              0.

    COMPONENTS OF ACCELERATION DUE TO GRAVITY VECTOR      (M/S**2) =  0.              0.              9.81000E+00

    EXTERNALLY APPLIED FORCE VECTOR ACTING ON CAGE             (N) =  0.              0.              0.
    CORRESPONDING POSITION VECTOR IN INERTIAL FRAME            (M) =  1.30000E-02     0.              0.
```

MATERIAL PROPERTIES AND INERTIAL PARAMETERS

		ROLLING ELEMENT	CAGE	RACEWAYS	SHAFT	HOUSING
ELASTIC MODULUS	(N/M**2)	1.99948E+11	3.44738E+09	1.99948E+11	1.99948E+11	1.99948E+11
POISSON-S RATIO		2.50000E-01	4.50000E-01	2.50000E-01	2.50000E-01	2.50000E-01
MASS DENSITY	(KGM/M**3)	7.75037E+03	1.19854E+03	7.75037E+03	7.75037E+03	7.75037E+03

		ROLLING ELEMENT	CAGE	OUTER RACE	INNER RACE
MASS	(KGM)	1.62354E-02	7.55796E-02	9.33017E-01	7.15787E-01
MOM OF INER -X	(KGM*M**2)	4.09157E-07	2.97122E-04	4.68519E-03	2.12084E-03
MOM OF INER -Y	(KGM*M**2)	4.09157E-07	1.52819E-04	2.42097E-03	1.12055E-03
MOM OF INER -Z	(KGM*M**2)	4.09157E-07	1.52819E-04	2.42097E-03	1.12055E-03

BALL/RACE DAMPING RATIO = 0.
BALL/CAGE DAMPING RATIO = 0.
RACE/CAGE DAMPING RATIO = 0.

SCALE FACTORS, INTEGRATION DETAILS AND OUTPUT CONTROLS

LENGTH SCALE (M) = 7.93750E-03	MIN STEP SIZE = 1.00000E-03	INITIAL STEP SIZE = 2.00000E-01
LOAD SCALE (N) = 1.78000E+02	MAX STEP SIZE = 1.00000E+01	TRUNCATION LIMIT = 1.00000E-04
TIME SCALE (S) = 8.50870E-04	FINAL TIME = 2.00000E+03	STEP OPT CODES = 6 50 1
DATA MONITOR CODE =500	PRINT CODES = 5200 5	AUTO PLOT CODES = 1 20
INT METHOD CODE = 0	SOLUTION MODE = 4	

OUTPUT FROM USER ACCESSIBLE ROUTINES ---
****

STEP NO 1   TAU = 0.   TIME = 0. S   100MM-DMA-BRG IRG-CAGE 60RPM 178N**

ROLLING ELEMENT PARAMETERS

RE NO	CONTACT ANGLES (DEG) OUTER RACE	OUTER RACE	INNER RACE	INNER RACE	CONTACT LOADS (N) OUTER RACE	INNER RACE	CONTACT DEFLS (M) OUTER RACE	INNER RACE	CONTACT STRESSES (N/M**2) OUTER RACE	INNER RACE
1	2.467E+01	0.	2.468E+01	0.	2.245E+01	2.244E+01	7.387E-07	7.550E-07	3.752E+08	4.168E+08
7	2.467E+01	0.	2.468E+01	0.	2.245E+01	2.244E+01	7.387E-07	7.550E-07	3.752E+08	4.168E+08
13	2.467E+01	0.	2.468E+01	0.	2.245E+01	2.244E+01	7.387E-07	7.550E-07	3.752E+08	4.168E+08
19	2.467E+01	0.	2.468E+01	0.	2.245E+01	2.244E+01	7.387E-07	7.550E-07	3.752E+08	4.168E+08

RE NO	CONTACT HALF WIDTHS (M) OUTER RACE	INNER RACE	OUTER RACE	INNER RACE	LOAD*SLIP INTEGRALS (N*M/S) OUTER RACE	INNER RACE	TRAC*SLIP INTEGRALS (N*M/S) OUTER RACE	INNER RACE	SPIN/ROLL RATIOS OUTER RACE	INNER RACE
1	4.663E-04	4.749E-04	6.125E-05	5.414E-05	1.031E-02	1.391E-03	2.829E-07	7.081E-09	-1.200E-01	0.
7	4.663E-04	4.749E-04	6.125E-05	5.414E-05	1.031E-02	1.391E-03	2.829E-07	7.081E-09	-1.200E-01	0.
13	4.663E-04	4.749E-04	6.125E-05	5.414E-05	1.031E-02	1.391E-03	2.829E-07	7.081E-09	-1.200E-01	0.
19	4.663E-04	4.749E-04	6.125E-05	5.414E-05	1.031E-02	1.391E-03	2.829E-07	7.081E-09	-1.200E-01	0.

RE NO	ORB POS (DEG)	MASS CENTER VELOCITIES (M/S) AXIAL	(M/S) RADIAL	(RPM) ORBITAL	ANGULAR VELOCITIES (RPM) X COMP	Y COMP	Z COMP	RE/CAGE CON FORCES (N) NORMAL	(N) TRACTION	(DEG) CON ANGLE
1	0.	0.	0.	2.654E+01	-2.060E+02	0.	1.100E+02	0.	0.	0.
7	1.137E+02	0.	0.	2.654E+01	-2.060E+02	0.	1.100E+02	0.	0.	1.800E+02
13	2.274E+02	0.	0.	2.654E+01	-2.060E+02	0.	1.100E+02	0.	0.	4.233E-13
19	3.411E+02	0.	0.	2.654E+01	-2.060E+02	0.	1.100E+02	0.	0.	3.600E+02

RE NO	RE/CAGE MIN CLS (M)	RE/CAGE SLIP VELS (M/S)	RE/RACE SLIP VELOCITIES (M/S) OUTER RACE I	OUTER RACE II	INNER RACE I	INNER RACE II	RE/RACE TRACTION COEFFICIENTS OUTER RACE I	OUTER RACE II	INNER RACE I	INNER RACE II
1	1.500E-04	0.	0.	1.036E-15	0.	4.637E-05	0.	4.246E-17	0.	1.901E-06
7	1.080E-05	0.	0.	1.036E-15	0.	4.637E-05	0.	4.246E-17	0.	1.901E-06
13	3.817E-05	0.	0.	1.036E-15	0.	4.637E-05	0.	4.246E-17	0.	1.901E-06
19	1.006E-04	0.	0.	1.036E-15	0.	4.637E-05	0.	4.246E-17	0.	1.901E-06

```
STEP NO       1       TAU =  0.            TIME =  0.        S         100MM-DMA-BRG  IRG-CAGE  60RPM  178N**
==============       ================      ===================

RACE AND CAGE PARAMETERS
------------------------

            **** MASS CENTER POSITIONS ***** **** MASS CENTER VELOCITIES **** ***** ANGULAR  POSITIONS *******
                   (M)          (M)       (DEG)       (M/S)        (M/S)        (RPM)                  (DEG)
                  AXIAL        RADIAL    ORBITAL      AXIAL       RADIAL       ORBITAL      X COMP     Y COMP     Z COMP

       CAGE -6.364E-06  1.520E-04  0.         0.          0.           0.           0.         0.         0.
 OUTER RACE  0.         0.         0.         0.          0.           0.           0.         0.         0.
 INNER RACE -1.268E-05  0.         0.         0.          0.           0.           0.         0.         0.

            ****** ANGULAR VELOCITIES ****** ******** NET ACC FORCES ******** ******** NET ACC MOMENTS *******
                          (RPM)                                  (N)                                  (N*M)
             X COMP      Y COMP      Z COMP      X COMP      Y COMP      Z COMP      X COMP      Y COMP      Z COMP

       CAGE  2.654E+01  0.          0.          0.         -7.414E-02 -7.457E+00 -6.018E-04  0.          2.165E-03
 OUTER RACE  0.         0.          0.          1.780E+02  5.810E-12 -4.348E-12  7.254E-05 -9.788E-14 -8.910E-14
 INNER RACE  6.000E+01  0.          0.         -1.780E+02 -5.850E-12  3.715E-12 -2.952E-05  9.537E-14  5.396E-14

           ************ RACE/CAGE FORCES ************* RACE/CAGE  RACE/CAGE  EFFECTIVE
                  (N)          (N)        (DEG)       (DEG)       MIN CLS   SLIP VELS   DIA PLAY
                NORMAL      TRACTION   CON ANGLE   ATT ANGLE      (M)        (M/S)        (M)

 LAND NO  1  0.          0.          0.          0.          4.443E-05  0.          3.929E-04
 LAND NO  2  0.          0.          0.          0.          4.443E-05  0.          3.929E-04


APPLIED PARAMETERS
------------------

            ************ FORCES ************ ************ MOMENTS ***********
                               (N)                                  (N*M)
             X COMP      Y COMP      Z COMP      X COMP      Y COMP      Z COMP

 OUTER RACE -1.780E+02 -5.810E-12  4.348E-12 -7.254E-05  9.788E-14  8.910E-14
 INNER RACE  1.780E+02  5.850E-12 -3.715E-12  2.952E-05 -9.537E-14 -5.396E-14

            ** MASS CENTER ACCELERATIONS *** **** ANGULAR ACCELERATIONS *****
                      (M/S**2)                             (RPM/S)
             X COMP      Y COMP      Z COMP      X COMP      Y COMP      Z COMP

 OUTER RACE  0.          0.          0.          0.          0.          0.
 INNER RACE  0.          0.          0.          0.          0.          0.

     NET BRG LOSS       (N*M/S) =   1.763E-02          NET LOAD*SLIP     (N*M/S) =   1.985E+00
     CURRENT LIFE       (HOURS) =   2.926E+13          OUTER RACE FIT        (M) =   1.000E-05
     INTERNAL CLEARANCE     (M) =   1.132E-04          INNER RACE FIT        (M) =   1.000E-05
     O.R. HOOP         (N/M**2) =  -1.158E+07          I.R. HOOP        (N/M**2) =   1.434E+07
     CAGE HOOP         (N/M**2) =   4.059E+01
```

```
RRRRRRRRRRRRRRR    DDDDDDDDDDDDDDD    RRRRRRRRRRRRRRR    EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
RRRRRRRRRRRRRRRR   DDDDDDDDDDDDDDDD   RRRRRRRRRRRRRRRR   EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBBB
   RR        RR       DD        DD       RR        RR    EE                     BB        BB
   RR        RR       DD        DD       RR        RR    EE                     BB        BB
   RRRRRRRRRRRRR      DD        DD       RRRRRRRRRRRRR   EEEEEEEEEE             BBBBBBBBBBBB
   RRRRRRRRRRRR       DD        DD       RRRRRRRRRRRR    EEEEEEEEEE             BBBBBBBBBBBB
   RR      RR         DD        DD       RR      RR      EE                     BB        BB
   RR        RR       DD        DD       RR        RR    EE                     BB        BB
   RR         RR   DDDDDDDDDDDDDDDD      RR         RR   EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBBB
   RR          RR  DDDDDDDDDDDDDDD       RR          RR  EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
```

R A P I D   S I M U L A T I O N   O F   T H E

D Y N A M I C S   O F   R O L L I N G   E L E M E N T   B E A R I N G S

-A REAL TIME PERFORMANCE SIMULATION-
(VERSION RAPIDREB.0)

PRADEEP K. GUPTA
MECHANICAL TECHNOLOGY INCORPORATED
968 ALBANY-SHAKER ROAD
LATHAM, NEW YORK
U. S. A.

BEARING TYPE -- BALL   SPEC CODE -- 150MM-DMA-BRG BG-CAGE 62.5RPM 534N***

BEARING GEOMETRY

BORE	(M) =	1.50000E-01		OUTER RACE SHRINK FIT	(M) =	1.00000E-05
OUTSIDE DIAMETER	(M) =	2.25000E-01		INNER RACE SHRINK FIT	(M) =	1.00000E-05
SHAFT INNER DIA	(M) =	1.37000E-01		HOUSING OUTER DIA	(M) =	2.44000E-01
BALL DIAMETER	(M) =	2.22250E-02		NUMBER OF BALLS	=	16
PITCH DIAMETER	(M) =	1.87500E-01		OUTER RACE CUR FACTOR	=	5.20000E-01
CONTACT ANGLE	(DEG) =	1.80000E+01		INNER RACE CUR FACTOR	=	5.30000E-01
END PLAY	(M) =	6.86790E-04		DIAMETRAL PLAY	(M) =	1.08777E-04
CAGE OUTER DIA	(M) =	1.97104E-01		CAGE OUTER DIA CLS	(M) =	3.98780E-03
CAGE INNER DIA	(M) =	1.77165E-01		CAGE INNER DIA CLS	(M) =	4.28600E-04
EFF CAGE WIDTH	(M) =	3.35280E-02		DIA RE/CAGE CLEARANCE	(M) =	4.06400E-04
RE/CAGE CONE ANG	(DEG) =	3.75000E+01		RE/CAGE CONE HEIGHT	(M) =	-3.18353E-03

LUBRICATION DETAILS
----

1. ROLLING ELEMENT/CAGE AND RACE/CAGE PARAMETERS ---

	INTERACTION CODE	LUB VISCOCITY (N*S/M**2)	MAX FILM (M)
ROLLING ELEMENT/CAGE INTERACTION	0	0.	1.00000E-04
RACE/CAGE INTERACTION	0	0.	

RE/CAGE DRY CONTACT TRACTION PARAMETERS --

```
CRIT FILM THICKNESS  (M)   =  2.50000E-07
COEFFICIENT A              =  1.00000E-02     COEFFICIENT B    1/ (M/S) =  0.
COEFFICIENT C     1/ (M/S) =  0.              COEFFICIENT D             = -1.00000E-02
```

2. ROLLING ELEMENT/RACE PARAMETERS (TRACTION CODE = 0) ---

```
COEFFICIENT A              =  0.              COEFFICIENT B    1/ (M/S) =  4.10000E-02
COEFFICIENT C     1/ (M/S) =  0.              COEFFICIENT D             = -1.00000E-02
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APPLIED LOADS AND SPEEDS
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 1. QUASI-STATIC SIMULATION ---

    AXIAL  LOAD            (N) =  5.34000E+02      OUTER RACE ANG VEL (RPM) =  0.
    RADIAL DISPLACEMENT    (M) =  0.               INNER RACE ANG VEL (RPM) =  6.25000E+01
    RELATIVE MISALIGNMENT (DEG) = 0.

 2. DYNAMIC SIMULATION ---

                   *************** LOADS   (N) ************      ******** MASS CEN ACC   (M/S**2) ********
                             I              II             III                 I              II              III

    OUTER RACE                                                   0.             0.             0.
    INNER RACE                                                   0.             0.             0.

                   ************ MOMENTS    (N*M) ***********      ******** ANG ACCELERATION (RPM/S) *******
                             I              II             III                 I              II              III

    OUTER RACE                                                   0.             0.             0.
    INNER RACE                                                   0.             0.             0.

                   ******** ANGULAR VELOCITIES (RPM)********
                             I              II             III

    OUTER RACE     0.             0.             0.
    INNER RACE     6.25000E+01    0.             0.

    COMPONENTS OF ACCELERATION DUE TO GRAVITY VECTOR      (M/S**2) = 0.             0.             9.81000E+00

    EXTERNALLY APPLIED FORCE VECTOR ACTING ON CAGE             (N) = 0.             0.             0.
    CORRESPONDING POSITION VECTOR IN INERTIAL FRAME            (M) = 1.67640E-02    0.             0.
```

MATERIAL PROPERTIES AND INERTIAL PARAMETERS

		ROLLING ELEMENT	CAGE	RACEWAYS	SHAFT	HOUSING
ELASTIC MODULUS	(N/M**2)	1.99948E+11	3.44738E+09	1.99948E+11	1.99948E+11	1.99948E+11
POISSON-S RATIO		2.50000E-01	4.50000E-01	2.50000E-01	2.50000E-01	2.50000E-01
MASS DENSITY	(KGM/M**3)	7.75037E+03	1.19854E+03	7.75037E+03	7.75037E+03	7.75037E+03

		ROLLING ELEMENT	CAGE	OUTER RACE	INNER RACE
MASS	(KGM)	4.45499E-02	1.81437E-01	3.02448E+00	2.33032E+00
MOM OF INER -X	(KGM*M**2)	2.20054E-06	1.64186E-03	3.40526E-02	1.56168E-02
MOM OF INER -Y	(KGM*M**2)	2.20054E-06	8.38715E-04	1.75243E-02	8.19210E-03
MOM OF INER -Z	(KGM*M**2)	2.20054E-06	8.38715E-04	1.75243E-02	8.19210E-03

BALL/RACE DAMPING RATIO = 0.
BALL/CAGE DAMPING RATIO = 0.
RACE/CAGE DAMPING RATIO = 0.

SCALE FACTORS, INTEGRATION DETAILS AND OUTPUT CONTROLS

LENGTH SCALE (M) = 1.11125E-02	MIN STEP SIZE = 1.00000E-03	INITIAL STEP SIZE = 2.00000E-01
LOAD SCALE (N) = 5.34000E+02	MAX STEP SIZE = 1.00000E+01	TRUNCATION LIMIT = 1.00000E-04
TIME SCALE (S) = 9.62850E-04	FINAL TIME = 2.00000E+03	STEP OPT CODES = 6 10 1
DATA MONITOR CODE =500	PRINT CODES = 5200 5	AUTO PLOT CODES = 1 17
INT METHOD CODE = 0	SOLUTION MODE = 4	

OUTPUT FROM USER ACCESSIBLE ROUTINES ---
****

STEP NO 1 TAU = 0. TIME = 0. S 150MM-DMA-BRG BG-CAGE 62.5RPM 534N***

ROLLING ELEMENT PARAMETERS

RE NO	CONTACT ANGLES (DEG) OUTER RACE	OUTER RACE	INNER RACE	INNER RACE	CONTACT LOADS (N) OUTER RACE	INNER RACE	CONTACT DEFLS (M) OUTER RACE	INNER RACE	CONTACT STRESSES (N/M**2) OUTER RACE	INNER RACE
1	1.779E+01	0.	1.779E+01	0.	1.093E+02	1.092E+02	1.897E-06	2.124E-06	5.087E+08	6.123E+08
7	1.779E+01	0.	1.779E+01	0.	1.093E+02	1.092E+02	1.897E-06	2.124E-06	5.087E+08	6.123E+08
13	1.779E+01	0.	1.779E+01	0.	1.093E+02	1.092E+02	1.897E-06	2.124E-06	5.087E+08	6.123E+08

RE NO	CONTACT HALF WIDTHS (M) OUTER RACE	INNER RACE	OUTER RACE	INNER RACE	LOAD*SLIP INTEGRALS (N*M/S) OUTER RACE	INNER RACE	TRAC*SLIP INTEGRALS (N*M/S) OUTER RACE	INNER RACE	SPIN/ROLL RATIOS OUTER RACE	INNER RACE
1	8.842E-04	7.673E-04	1.160E-04	1.110E-04	1.489E-02	6.293E-02	1.668E-07	2.354E-06	2.942E-16	6.516E-02
7	8.842E-04	7.673E-04	1.160E-04	1.110E-04	1.489E-02	6.293E-02	1.668E-07	2.354E-06	2.942E-16	6.516E-02
13	8.842E-04	7.673E-04	1.160E-04	1.110E-04	1.489E-02	6.293E-02	1.668E-07	2.354E-06	2.942E-16	6.516E-02

RE NO	ORB POS (DEG)	MASS CENTER VELOCITIES (M/S) AXIAL	(M/S) RADIAL	(RPM) ORBITAL	ANGULAR VELOCITIES (RPM) X COMP	Y COMP	Z COMP	RE/CAGE CON FORCES (N) NORMAL	(N) TRACTION	(DEG) CON ANGLE
1	0.	0.	0.	2.772E+01	-2.505E+02	0.	7.146E+01	0.	0.	0.
7	1.350E+02	0.	0.	2.772E+01	-2.505E+02	0.	7.146E+01	0.	0.	1.800E+02
13	2.700E+02	0.	0.	2.772E+01	-2.505E+02	0.	7.146E+01	2.705E-01	2.705E-03	3.600E+02

RE NO	RE/CAGE MIN CLS (M)	RE/CAGE SLIP VELS (M/S)	RE/RACE SLIP VELOCITIES (M/S) OUTER RACE I	OUTER RACE II	INNER RACE I	INNER RACE II	RE/RACE TRACTION COEFFICIENTS OUTER RACE I	OUTER RACE II	INNER RACE I	INNER RACE II
1	2.032E-04	0.	0.	-1.020E-04	0.	0.	0.	-4.181E-06	0.	0.
7	5.952E-05	0.	0.	-1.020E-04	0.	0.	0.	-4.181E-06	0.	0.
13	2.467E-18	3.348E-01	0.	-1.020E-04	0.	0.	0.	-4.181E-06	0.	0.

RE NO	RE/CAGE GUIDANCE PARAMETERS NOR LOAD (N)	FRIC LOAD (N)	MIN FILM (M)	SLIP VEL (M/S)	CON ANGLE (DEG)
1	0.	0.	1.750E-04	0.	0.
7	0.	0.	1.504E-04	0.	1.800E+02
13	0.	0.	4.787E-05	0.	3.823E-12

STEP NO 1 TAU = 0. TIME = 0. S 150MM-DMA-BRG BG-CAGE 62.5RPM 534N***

RACE AND CAGE PARAMETERS

	MASS CENTER POSITIONS (M) AXIAL	(M) RADIAL	(DEG) ORBITAL	MASS CENTER VELOCITIES (M/S) AXIAL	(M/S) RADIAL	(RPM) ORBITAL	ANGULAR POSITIONS (DEG) X COMP	Y COMP	Z COMP
CAGE	-1.002E-06	2.032E-04	0.	0.	0.	0.	0.	0.	0.
OUTER RACE	0.	0.	0.	0.	0.	0.	0.	0.	0.
INNER RACE	-2.661E-06	0.	0.	0.	0.	0.	0.	0.	0.

	ANGULAR VELOCITIES (RPM) X COMP	Y COMP	Z COMP	NET ACC FORCES (N) X COMP	Y COMP	Z COMP	NET ACC MOMENTS (N*M) X COMP	Y COMP	Z COMP
CAGE	2.772E+01	0.	0.	0.	-5.241E-03	1.239E+00	-5.930E-05	0.	1.260E-04
OUTER RACE	0.	0.	0.	5.340E+02	3.107E-11	-2.561E-11	5.024E-04	-6.641E-13	-5.692E-13
INNER RACE	6.250E+01	0.	0.	-5.340E+02	-2.869E-11	2.324E-11	-9.485E-04	6.325E-13	7.379E-13

	RACE/CAGE FORCES (N) NORMAL	(N) TRACTION	(DEG) CON ANGLE	(DEG) ATT ANGLE	RACE/CAGE MIN CLS (M)	RACE/CAGE SLIP VELS (M/S)	EFFECTIVE DIA PLAY (M)
LAND NO 1	0.	0.	0.	0.	0.	0.	0.
LAND NO 2	0.	0.	0.	0.	0.	0.	0.

APPLIED PARAMETERS

	FORCES (N) X COMP	Y COMP	Z COMP	MOMENTS (N*M) X COMP	Y COMP	Z COMP
OUTER RACE	-5.340E+02	-3.107E-11	2.561E-11	-5.024E-04	6.641E-13	5.692E-13
INNER RACE	5.340E+02	2.869E-11	-2.324E-11	9.485E-04	-6.325E-13	-7.379E-13

	MASS CENTER ACCELERATIONS (M/S**2) X COMP	Y COMP	Z COMP	ANGULAR ACCELERATIONS (RPM/S) X COMP	Y COMP	Z COMP
OUTER RACE	0.	0.	0.	0.	0.	0.
INNER RACE	0.	0.	0.	0.	0.	0.

NET BRG LOSS	(N*M/S) =	1.852E-03	NET LOAD*SLIP	(N*M/S) =	1.426E+00
CURRENT LIFE	(HOURS) =	6.850E+11	OUTER RACE FIT	(M) =	1.000E-05
INTERNAL CLEARANCE	(M) =	9.861E-05	INNER RACE FIT	(M) =	1.000E-05
O.R. HOOP	(N/M**2) =	-5.098E+06	I.R. HOOP	(N/M**2) =	6.457E+06
CAGE HOOP	(N/M**2) =	9.549E+01			

APPENDIX C

TYPICAL COMPUTER OUTPUT FOR THE 100 mm ENGINE BEARING

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RRRRRRRRRRRRRRR    DDDDDDDDDDDDDDD    RRRRRRRRRRRRRRR    EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
RRRRRRRRRRRRRRRR   DDDDDDDDDDDDDDDD   RRRRRRRRRRRRRRRR   EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBBB
  RR         RR      DD        DD       RR         RR    EE                    BB          BB
  RR         RR      DD        DD       RR         RR    EE                    BB          BB
  RRRRRRRRRRRRR      DD        DD       RRRRRRRRRRRRR    EEEEEEEEEE            BBBBBBBBBBBB
  RRRRRRRRRRRR       DD        DD       RRRRRRRRRRRR     EEEEEEEEEE            BBBBBBBBBBBB
  RR      RR         DD        DD       RR       RR      EE                    BB          BB
  RR       RR        DD        DD       RR        RR     EE                    BB          BB
  RR         RR    DDDDDDDDDDDDDDDD     RR         RR    EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBBB
  RR          RR   DDDDDDDDDDDDDDD      RR          RR   EEEEEEEEEEEEEEEE    BBBBBBBBBBBBBBB
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R A P I D   S I M U L A T I O N   O F   T H E

D Y N A M I C S   O F   R O L L I N G   E L E M E N T   B E A R I N G S

-A REAL TIME PERFORMANCE SIMULATION-
(VERSION RAPIDREB.0)

PRADEEP K. GUPTA
MECHANICAL TECHNOLOGY INCORPORATED
968 ALBANY-SHAKER ROAD
LATHAM, NEW YORK
U. S. A.

BEARING TYPE -- BALL   SPEC CODE -- 100MM ENGINE BRG MOD CAGE 18/4.5KN 20KRPM

BEARING GEOMETRY

BORE	(M) =	1.00000E-01	OUTER RACE SHRINK FIT	(M) =	1.00000E-05
OUTSIDE DIAMETER	(M) =	1.80000E-01	INNER RACE SHRINK FIT	(M) =	5.00000E-05
SHAFT INNER DIA	(M) =	2.00000E-02	HOUSING OUTER DIA	(M) =	2.05000E-02
BALL DIAMETER	(M) =	1.90500E-02	NUMBER OF BALLS	=	18
PITCH DIAMETER	(M) =	1.40000E-01	OUTER RACE CUR FACTOR	=	5.20000E-01
CONTACT ANGLE	(DEG) =	2.50000E+01	INNER RACE CUR FACTOR	=	5.40000E-01
END PLAY	(M) =	9.66105E-04	DIAMETRAL PLAY	(M) =	2.14180E-04
CAGE OUTER DIA	(M) =	1.48800E-01	CAGE OUTER DIA CLS	(M) =	7.00000E-03
CAGE INNER DIA	(M) =	1.29700E-01	CAGE INNER DIA CLS	(M) =	3.00000E-03
EFF CAGE WIDTH	(M) =	2.73000E-02	DIA RE/CAGE CLEARANCE	(M) =	8.26000E-04

	GUIDANCE TYPE	GUIDING RACE RADIUS (M)	GUIDING CAGE RADIUS (M)	EFF LAND WIDTH (M)	CAGE HALF WIDTH (M)
GUIDING LAND I	2	6.42350E-02	6.48500E-02	4.00000E-03	1.36500E-02
GUIDING LAND II	2	6.42350E-02	6.48500E-02	4.00000E-03	1.36500E-02

LUBRICATION DETAILS

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LUBRICATION DETAILS
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 1. ROLLING ELEMENT/CAGE AND RACF/CAGE PARAMETERS ---

                                          INTERACTION  LUB VISCOCITY       MAX FILM
                                             CODE        (N*S/M**2)           (M)

    ROLLING ELEMENT/CAGE INTERACTION          1       7.12466F-03     4.00000E-04
    RACE/CAGE INTERACTION                     1       7.12466F-03

    RE/CAGE DRY CONTACT TRACTION PARAMETERS --

    CRIT FILM THICKNESS  (M)      =  5.00000E-07
    TRAC COEFF AT ZERO SLIP       =  0.                MAXIMUM TRAC COEFF            =  2.00000E-02
    TRAC COEFF AT INFINITE SLIP   =  1.60000E-02       SLIP AT MAX TRAC      (M/S)   =  1.00000E+00

    CAGE/RACE DRY CONTACT TRACTION PARAMETERS --

    CRIT FILM THICKNESS  (M)      =  5.00000E-07
    TRAC COEFF AT ZERO SLIP       =  0.                MAXIMUM TRAC COEFF            =  2.00000E-02
    TRAC COEFF AT INFINITE SLIP   =  1.60000E-02       SLIP AT MAX TRAC      (M/S)   =  1.00000E+00

 2. ROLLING ELEMENT/RACE PARAMETERS (TRACTION CODE = 3) ---

    FILM THICKNESS CODE           =            2       CRIT FILM THICKNESS     (M)   =  1.00000E-07
    INLET TEMP         (DEG-K)    =  3.30000E+02       PR-VIS COEFF       (M**2/N)   =  1.01447E-08
    INLET VIS       (N*S/M**2)    =  7.12466E-03       TEMP-VIS COEFF     (DEG-K)    =  2.85205E+03
    THERMAL COND   (N/S/DEG-C)    =  9.65788E-02       STARVATION PARAMETER          =  1.00000E+01

    VISCOCITY*      (N*S/M**2)    =  7.87989E-02       ROLLING SPEED EFFECT
    PR-VIS COEFF*     (M**2/N)    =  5.22136E-09       PARAMETERS -  VR      (M/S)   =  2.28600E+01
    TEMP-VIS COEFF*  (1/DEG-K)    =  5.46451E-02                     VN              = -7.03200E-01

    OUTER RACE TEMP    (DEG-K)    =  3.30000E+02       INNER RACE TEMP     (DEG-K)   =  3.30000E+02

 3. TRACTION PARAMETERS OUT OF LUB MODEL BOUNDS ---

    TRAC COEFF AT ZERO SLIP       =  0.                MAXIMUM TRAC COEFF            =  2.00000E-02
    TRAC COEFF AT INFINITE SLIP   =  1.60000E-02       SLIP AT MAX TRAC      (M/S)   =  1.00000E+00

 4. LUBRICANT DRAG AND CHURNING PARAMETERS ---

    EFF LUB VIS     (N*S/M**2)    =  7.12466E-03       EFF LUB DEN      (KGM/M**3)   =  1.00000E+01
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APPLIED LOADS AND SPEEDS

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 1. QUASI-STATIC SIMULATION ---

    AXIAL  LOAD                (N) =  1.80000E+04      OUTER RACE ANG VEL (RPM) =  0.
    RADIAL LOAD                (N) =  4.50000E+03      INNER RACE ANG VEL (RPM) =  2.00000E+04
    RELATIVE MISALIGNMENT (DEG) =  0.

 2. DYNAMIC SIMULATION ---
                 *************** LOADS      (N) ************       ******** MASS CEN ACC    (M/S**2) ********
                                I                II            III                 I                II              III

    OUTER RACE                                                          0.                0.              0.
    INNER RACE                                                          0.                0.              0.

                 ************ MOMENTS      (N*M) ***********       ******** ANG ACCELERATION (RPM/S) *******
                                I                II            III                 I                II              III

    OUTER RACE                                                          0.                0.              0.
    INNER RACE                                                          0.                0.              0.

                 ******** ANGULAR VELOCITIES (RPM)********
                                I                II            III

    OUTER RACE     0.               0.             0.
    INNER RACE     2.00000E+04      0.             0.

    COMPONENTS OF ACCELERATION DUE TO GRAVITY VECTOR       (M/S**2) =  0.             0.              9.81000E+00

    EXTERNALLY APPLIED FORCE VECTOR ACTING ON CAGE               (N) =  0.             0.              0.
    CORRESPONDING POSITION VECTOR IN INERTIAL FRAME              (M) =  1.36500E-02    0.              0.
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MATERIAL PROPERTIES AND INERTIAL PARAMETERS
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                              ROLLING ELEMENT       CAGE        RACEWAYS        SHAFT        HOUSING

ELASTIC MODULUS   (N/M**2)      1.99948E+11     2.00000E+11    1.99948E+11    1.99948E+11    1.99948E+11
POISSON-S RATIO                 2.50000E-01     2.50000E-01    2.50000E-01    2.50000E-01    2.50000E-01
MASS DENSITY    (KGM/M**3)      7.75037E+03     7.75037E+03    7.75037E+03    7.75037E+03    7.75037E+03

                              ROLLING ELEMENT       CAGE        OUTER RACE     INNER RACE

MASS                 (KGM)      2.80547E-02     4.70585E-01    2.32900E+00    1.62894E+00
MOM OF INER -X  (KGM*M**2)      1.01811E-06     2.29196E-03    1.59413E-02    5.50250E-03
MOM OF INER -Y  (KGM*M**2)      1.01811E-06     1.17521E-03    8.25241E-03    2.94830E-03
MOM OF INER -Z  (KGM*M**2)      1.01811E-06     1.17521E-03    8.25241E-03    2.94830E-03

BALL/RACE DAMPING RATIO   =   0.
BALL/CAGE DAMPING RATIO   =   0.
RACE/CAGE DAMPING RATIO   =   0.

SCALE FACTORS, INTEGRATION DETAILS AND OUTPUT CONTROLS
------------------------------------------------------

LENGTH SCALE  (M) =  9.52500E-03   MIN STEP SIZE =  2.00000E-02   INITIAL STEP SIZE =  1.00000E-01
LOAD SCALE    (N) =  1.80000E+04   MAX STEP SIZE =  1.00000E+01   TRUNCATION LIMIT  =  1.00000E-04
TIME SCALE    (S) =  1.21843E-04   FINAL TIME    =  4.00000E+02   STEP OPT CODES    =  6 50   2

DATA MONITOR CODE =200             PRINT CODES   =  5 50  5       AUTO PLOT CODES   =  1 19 21
INT METHOD CODE   =  0             SOLUTION MODE =  3

OUTPUT FROM USER ACCESSIBLE ROUTINES ---
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STEP NO 1 TAU = 0. TIME = 0. S 100MM ENGINE BRG MOD CAGE 18/4.5KN 20KRPM

ROLLING ELEMENT PARAMETERS

RE NO	CONTACT ANGLES (DEG) OUTER RACE	CONTACT ANGLES (DEG) OUTER RACE	CONTACT ANGLES (DEG) INNER RACE	CONTACT ANGLES (DEG) INNER RACE	CONTACT LOADS (N) OUTER RACE	CONTACT LOADS (N) INNER RACE	CONTACT DEFLS (M) OUTER RACE	CONTACT DEFLS (M) INNER RACE	CONTACT STRESSES (N/M**2) OUTER RACE	CONTACT STRESSES (N/M**2) INNER RACE
1	1.599E+01	0.	2.527E+01	0.	4.420E+03	2.852E+03	2.350E-05	2.098E-05	1.921E+09	2.147E+09
7	1.434E+01	0.	2.640E+01	-4.467E-03	3.599E+03	2.005E+03	2.049E-05	1.658E-05	1.793E+09	1.908E+09
13	1.434E+01	0.	2.640E+01	4.467E-03	3.599E+03	2.005E+03	2.049E-05	1.658E-05	1.793E+09	1.908E+09

RE NO	CONTACT HALF WIDTHS (M) OUTER RACE	CONTACT HALF WIDTHS (M) INNER RACE	CONTACT HALF WIDTHS (M) OUTER RACE	CONTACT HALF WIDTHS (M) INNER RACE	LOAD*SLIP INTEGRALS (N*M/S) OUTER RACE	LOAD*SLIP INTEGRALS (N*M/S) INNER RACE	TRAC*SLIP INTEGRALS (N*M/S) OUTER RACE	TRAC*SLIP INTEGRALS (N*M/S) INNER RACE	SPIN/ROLL RATIOS OUTER RACE	SPIN/ROLL RATIOS INNER RACE
1	2.880E-03	1.934E-03	3.815E-04	3.281E-04	2.099E+03	4.124E+03	3.039E+01	7.434E+01	-3.606E-15	2.311E-01
7	2.689E-03	1.719E-03	3.564E-04	2.919E-04	1.496E+03	3.055E+03	1.692E+01	5.088E+01	6.642E-16	2.754E-01
13	2.689E-03	1.719E-03	3.564E-04	2.919E-04	1.496E+03	3.055E+03	1.692E+01	5.088E+01	1.727E-15	2.754E-01

RE NO	ORB POS (DEG)	MASS CENTER VELOCITIES (M/S) AXIAL	MASS CENTER VELOCITIES (M/S) RADIAL	MASS CENTER VELOCITIES (RPM) ORBITAL	ANGULAR VELOCITIES (RPM) X COMP	ANGULAR VELOCITIES (RPM) Y COMP	ANGULAR VELOCITIES (RPM) Z COMP	RE/CAGE CON FORCES (N) NORMAL	RE/CAGE CON FORCES (N) TRACTION	RE/CAGE CON FORCES (DEG) CON ANGLE
1	0.	0.	0.	8.804E+03	-7.143E+04	0.	1.794E+04	2.149E-02	2.617E-02	2.700E+02
7	1.200E+02	0.	0.	8.859E+03	-7.232E+04	0.	1.622E+04	1.276E+00	2.217E-01	1.791E+02
13	2.400E+02	0.	0.	8.859E+03	-7.232E+04	0.	1.622E+04	1.276E+00	2.217E-01	8.611E-01

RE NO	RE/CAGE MIN CLS (M)	RE/CAGE SLIP VELS (M/S)	RE/RACE SLIP VELOCITIES (M/S) OUTER RACE I	RE/RACE SLIP VELOCITIES (M/S) OUTER RACE II	RE/RACE SLIP VELOCITIES (M/S) INNER RACE I	RE/RACE SLIP VELOCITIES (M/S) INNER RACE II	RE/RACE TRACTION COEFFICIENTS OUTER RACE I	RE/RACE TRACTION COEFFICIENTS OUTER RACE II	RE/RACE TRACTION COEFFICIENTS INNER RACE I	RE/RACE TRACTION COEFFICIENTS INNER RACE II
1	4.050E-04	1.790E+01	0.	-3.523E-01	0.	0.	0.	-2.000E-02	0.	0.
7	1.471E-04	7.392E+01	0.	-3.087E-01	-1.407E-03	-5.555E-13	0.	-2.000E-02	-1.405E-13	-1.405E-13
13	1.471E-04	7.392E+01	-3.439E-14	-3.087E-01	1.407E-03	0.	-2.228E-15	-2.000E-02	0.	0.

RE NO	RE/RACE ISO FILM (M) OUTER RACE	RE/RACE ISO FILM (M) INNER RACE	THERMAL*SIDE LEAKAGE FACTORS OUTER RACE	THERMAL*SIDE LEAKAGE FACTORS INNER RACE	DRAG FORCE (N)	CHURN MOM (N*M)	NET DRAG+ CHURN LOSS (N*M/S)
1	1.043E-06	9.395E-07	3.798E-01	3.783E-01	2.731E+00	2.880E-03	1.986E+02
7	1.063E-06	9.597E-07	3.818E-01	3.886E-01	2.766E+00	2.907E-03	2.023E+02
13	1.063E-06	9.597E-07	3.818E-01	3.886E-01	2.766E+00	2.907E-03	2.023E+02

STEP NO 1   TAU = 0.   TIME = 0. S   100MM ENGINE BRG MOD CAGE 18/4.5KN 20KRPM

RACE AND CAGE PARAMETERS

	**** MASS CENTER POSITIONS *****			**** MASS CENTER VELOCITIES ****			***** ANGULAR POSITIONS *******		
	(M) AXIAL	(M) RADIAL	(DEG) ORBITAL	(M/S) AXIAL	(M/S) RADIAL	(RPM) ORBITAL	(DEG) X COMP	Y COMP	Z COMP
CAGE	-5.760E-05	3.070E-04	0.	0.	0.	0.	0.	0.	0.
OUTER RACE	0.	0.	0.	0.	0.	0.	0.	0.	0.
INNER RACE	-3.747E-05	7.039E-06	0.	0.	0.	0.	0.	0.	0.

	****** ANGULAR VELOCITIES ****** (RPM)			******** NET ACC FORCES ******** (N)			******** NET ACC MOMENTS ******* (N*M)		
	X COMP	Y COMP	Z COMP	X COMP	Y COMP	Z COMP	X COMP	Y COMP	Z COMP
CAGE	8.841E+03	0.	0.	3.288E-03	-2.478E+00	-9.451E+00	1.161E-01	2.544E-02	2.983E-02
OUTER RACE	0.	0.	0.	1.800E+04	-1.932E+01	4.364E+03	-1.462E+01	1.312E+02	-2.829E-02
INNER RACE	2.000E+04	0.	0.	-1.800E+04	-3.820E+00	-4.498E+03	3.108E-01	-1.332E+02	-1.170E-01

	************ RACE/CAGE FORCES **************				RACE/CAGE	RACE/CAGE	EFFECTIVE
	(N) NORMAL	(N) TRACTION	(DEG) CON ANGLE	(DEG) ATT ANGLE	MIN CLS (M)	SLIP VELS (M/S)	DIA PLAY (M)
LAND NO 1	8.395E-02	1.753E+00	1.800E+02	-5.230E+01	2.781E-04	7.459E+01	1.156E-03
LAND NO 2	8.395E-02	1.753E+00	1.800E+02	-5.230E+01	2.781E-04	7.459E+01	1.156E-03

CAGE CHURN MOM (N*M) = 7.962E-02   CAGE NET CHURN LOSS (N*M/S) = 6.153E+02

APPLIED PARAMETERS

	************ FORCES ************ (N)			************ MOMENTS *********** (N*M)		
	X COMP	Y COMP	Z COMP	X COMP	Y COMP	Z COMP
OUTER RACE	-1.800E+04	1.932E+01	-4.364E+03	1.462E+01	-1.312E+02	2.829E-02
INNER RACE	1.800E+04	3.820E+00	4.498E+03	-3.108E-01	1.332E+02	1.170E-01

	** MASS CENTER ACCELERATIONS *** (M/S**2)			**** ANGULAR ACCELERATIONS ***** (RPM/S)		
	X COMP	Y COMP	Z COMP	X COMP	Y COMP	Z COMP
OUTER RACE	0.	0.	0.	0.	0.	0.
INNER RACE	0.	0.	0.	0.	0.	0.

NET BRG LOSS	(N*M/S) =	6.182E+03	NET LOAD*SLIP	(N*M/S) =	9.324E+04
CURRENT LIFE	(HOURS) =	1.536E+03	OUTER RACE FIT	(M) =	1.000E-05
INTERNAL CLEARANCE	(M) =	1.062E-04	INNER RACE FIT	(M) =	9.767E-07
O.R. HOOP	(N/M**2) =	-1.427E+07	I.R. HOOP	(N/M**2) =	1.183E+08
CAGE HOOP	(N/M**2) =	3.511E+07			

DATE
ILME